

# The Risk Structure of Interest Rates Revisited

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# Overview

- Significant efforts already made to explain credit spread levels utilizing structural models derived from Merton (1974)
  - Most suffer from the difficulty in matching tenor between a risky coupon bond and a risk free US Treasury bond as well as premium/discount issues
  - US Treasury as risk free also presents issues due to state tax exemption on interest and excess liquidity problems in on-the-run issues
    - Hull et al., 2004, find risk free is approximately Libor – 10 bps
  - Bond price data primarily used in monthly time series (no means for fixed tenor time series without significant interpolation assumptions)
- Little produced to date to explain term structures of credit spreads
- This work will examine how well structural models describe the term structure of credit spreads utilizing daily credit derivative price data and daily US\$ Libor swap rates as a proxy for risk free
  - Modified version of Merton (1974) – This presentation
  - Collin-Dufresne and Goldstein (2001) introduces stationary leverage ratios
  - Zhou (2001) introduces jumps in asset values
- Motivation
  - examine the dynamics of corporate capital structures, in particular, the dynamics of the likelihood of corporate defaults

# Preliminary Results

- Market credit spread levels from credit default swaps observed at 1, 2, 5, and 10 year tenors explained well statistically by Modified Merton model spreads and US\$ Libor swap spreads
  - Well known drawback of low Merton spreads at short tenors and high quality reproduced (motivation for examining Zhou, 2001)
  - Negative relation between credit spread levels and level of risk free rates first pointed out by Duffee (1999) with bond spreads to Treasury holds at all tenors
- Market credit spread slopes between 1-5 year, 2-5 year, 2-10 year, 5-10 year explained well statistically by similar Modified Merton model spread slopes and US\$ Libor swap spread slopes
  - Negative slopes begin at much higher credit quality under Merton than in the market (motivation for examining Collin-Dufresne and Goldstein, 2001)
  - Risk free spread slope remains in strong negative relation to credit spread slope

# Review of Merton Model (1974)

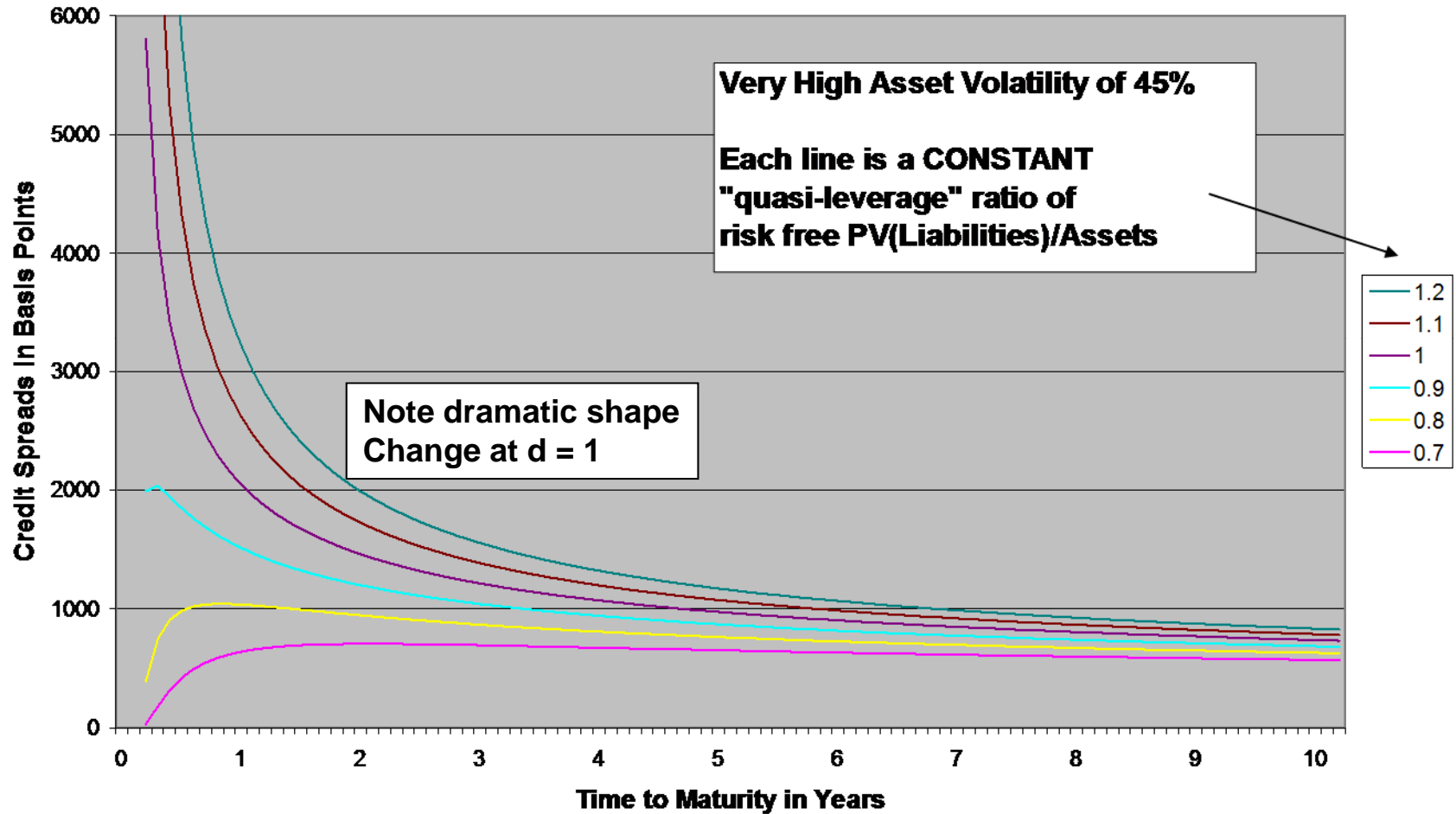
$$\text{Credit Spread} = \frac{-1}{T} \bullet \text{Ln} \left\{ \text{Norm} \left( - \left( \frac{0.5\sigma^2 T - \text{Ln}(d)}{\sigma\sqrt{T}} \right) \right) + \frac{1}{d} * \text{Norm} \left( - \left( \frac{0.5\sigma^2 T + \text{Ln}(d)}{\sigma\sqrt{T}} \right) \right) \right\}$$

$\sigma \equiv$  Volatility of Assets

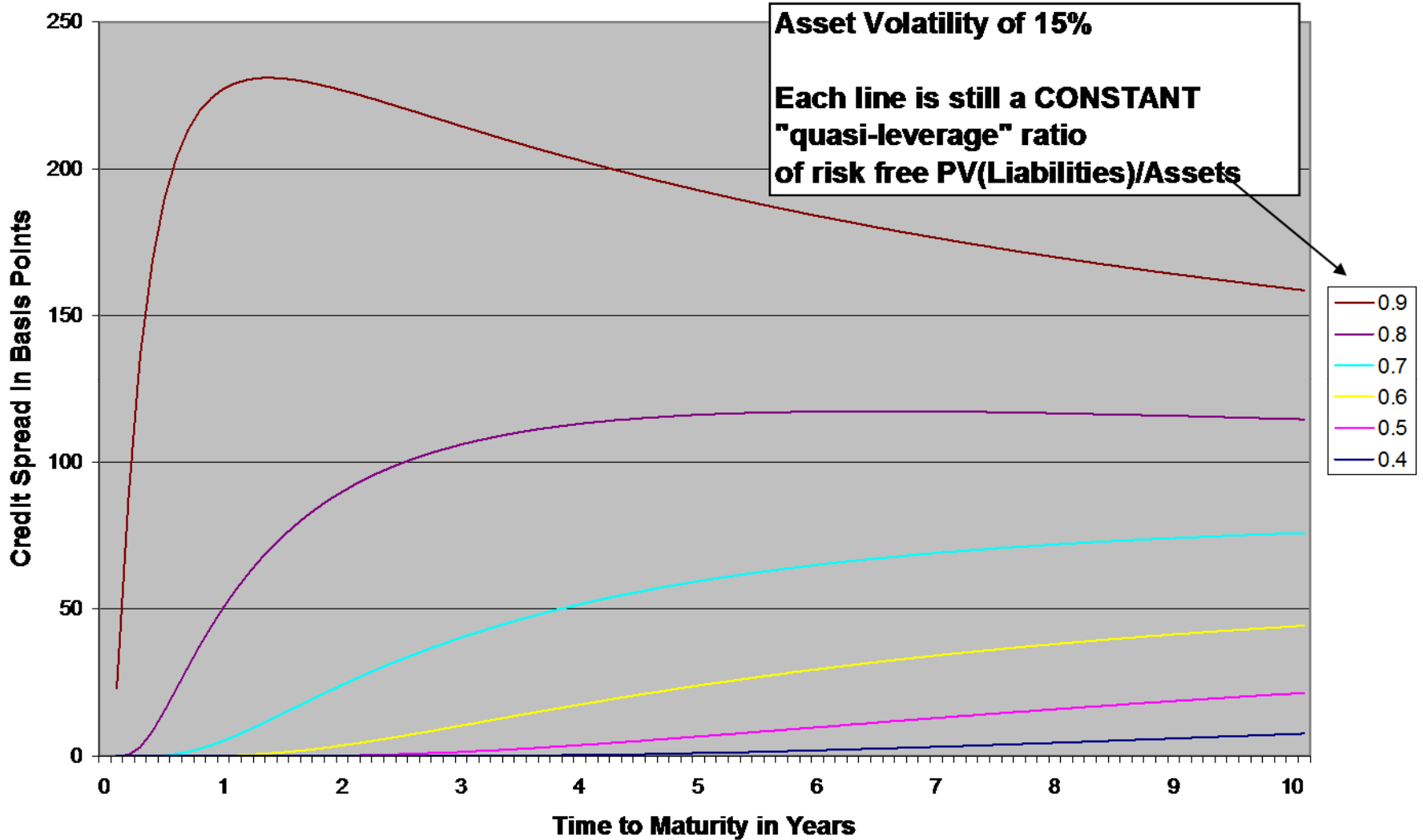
$$\text{Quasi- Leverage Ratio, } d \equiv \frac{B(T)e^{-rT}}{V} \equiv \frac{\text{Risk Free PV(Liabilities)}}{\text{Market Value of Assets}}$$

- Model expressed in terms of unobservable parameters for market assets
- These will need to be inferred from market equity
- I assume quasi-leverage ratio is independent of debt's time to maturity:  $B(T) = B_0 e^{rT}$ , so that  $d = B_0/V$ .
- This assumption approximates accrual or payment of coupon interest on par debt. It equates the risk-neutral expected growth of assets and liabilities.

## Merton Model as Depicted in Second Correction of Pitts and Selby (1983)



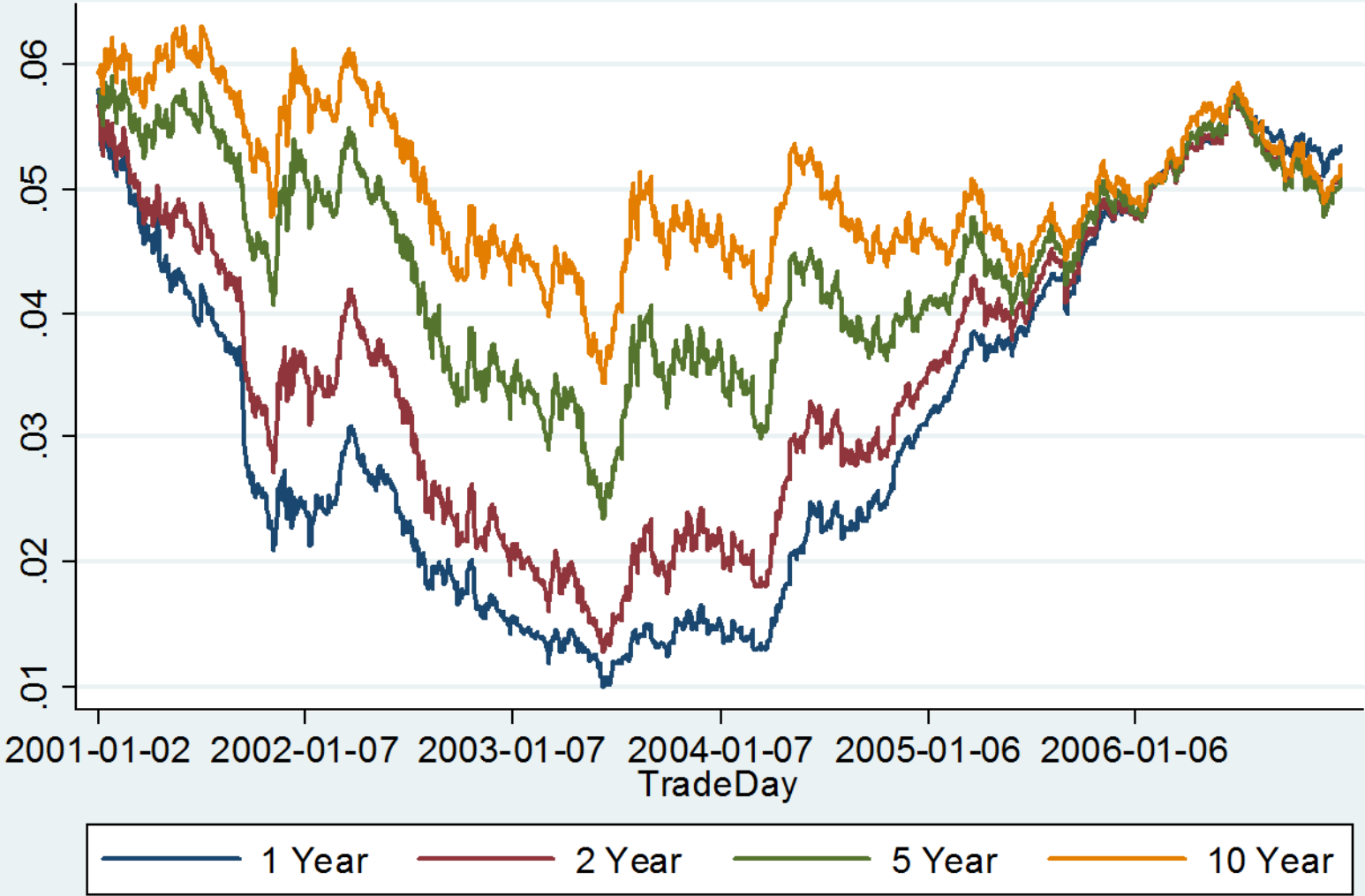
# Merton Model Parameterized (Approx) by Actual Data



# The Data

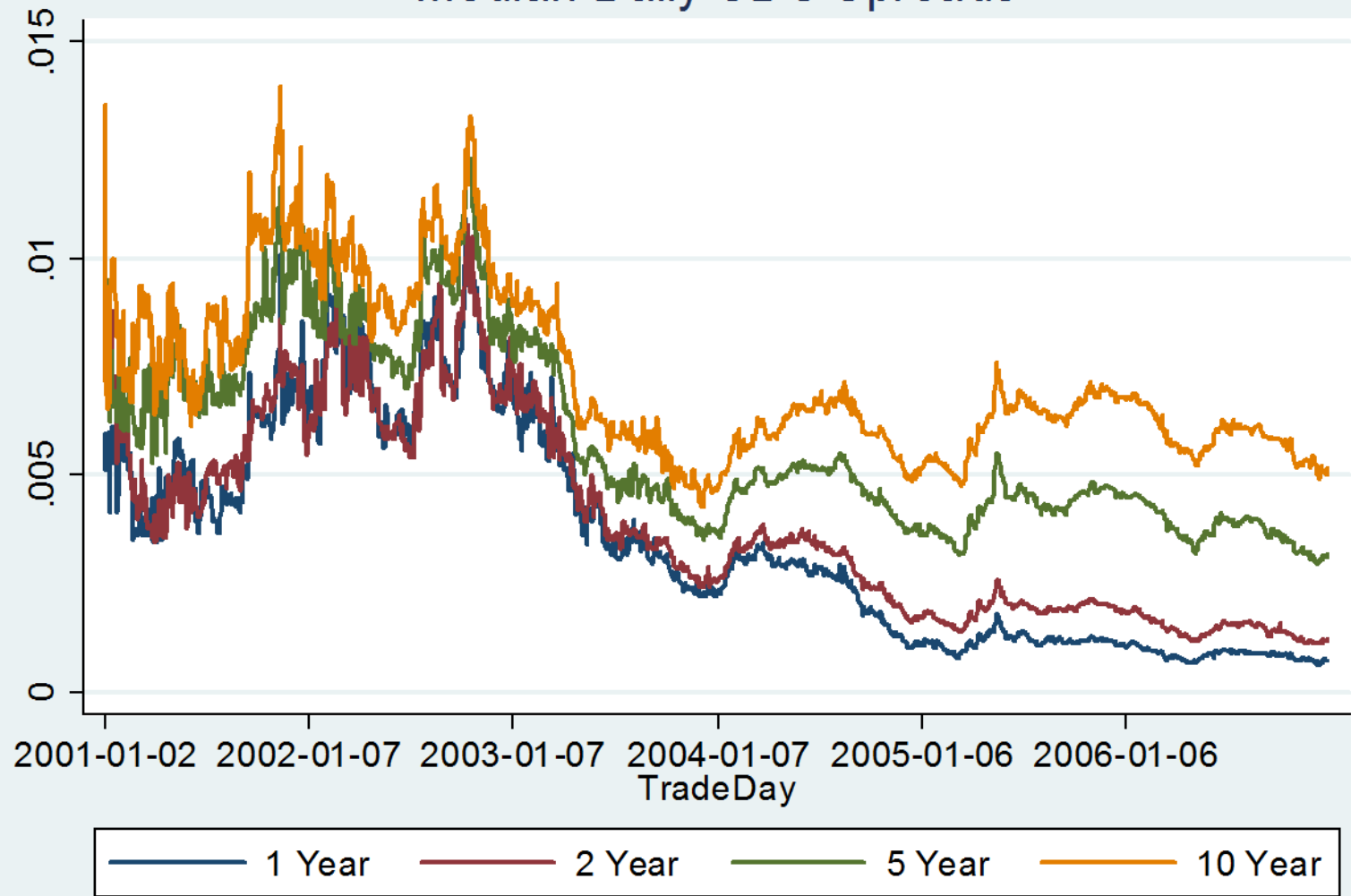
- Markit Partners Credit Default Swap data
  - Daily, senior unsecured, modified restructuring convention observations provided by minimum of 3 dealers quoting
  - Selected 1, 2, 5, and 10 year tenors (5 yr most prevalent)
  - From begin of 2001 through end of 2006
  - Approximately 200 firms in this sub-sample (letters A-D) producing approximately 180,000 firm-trading day observations
- Daily firm market equity value produced from CRSP
- Daily firm 91 ATM standardized implied equity volatility from OptionMetrics
- Daily senior unsecured S&P and Moody's credit ratings aggregated by firm from issue ratings in Mergent's Fixed Income Securities Database
- Quarterly Book Assets and Common Equity as of release date of quarterly earnings from Compustat Quarterly file
- Daily US\$ Libor Swap Rates from FRB Database

# Swap Rates





## Median Daily CDS Spreads



# Calculating Asset Value and Volatility

Two equations in two unknowns first proposed in Marcus and Shaked, 1984

$$1. \sigma_{\text{Equity}} \left( \frac{\text{Equity}}{B_0} \right) = N(d_1) \sigma_{\text{Assets}} \left( \frac{\text{Assets}}{B_0} \right)$$

$$2. \left( \frac{\text{Equity}}{B_0} \right) = \left( \frac{\text{Assets}}{B_0} \right) N(d_1) - N(d_2)$$

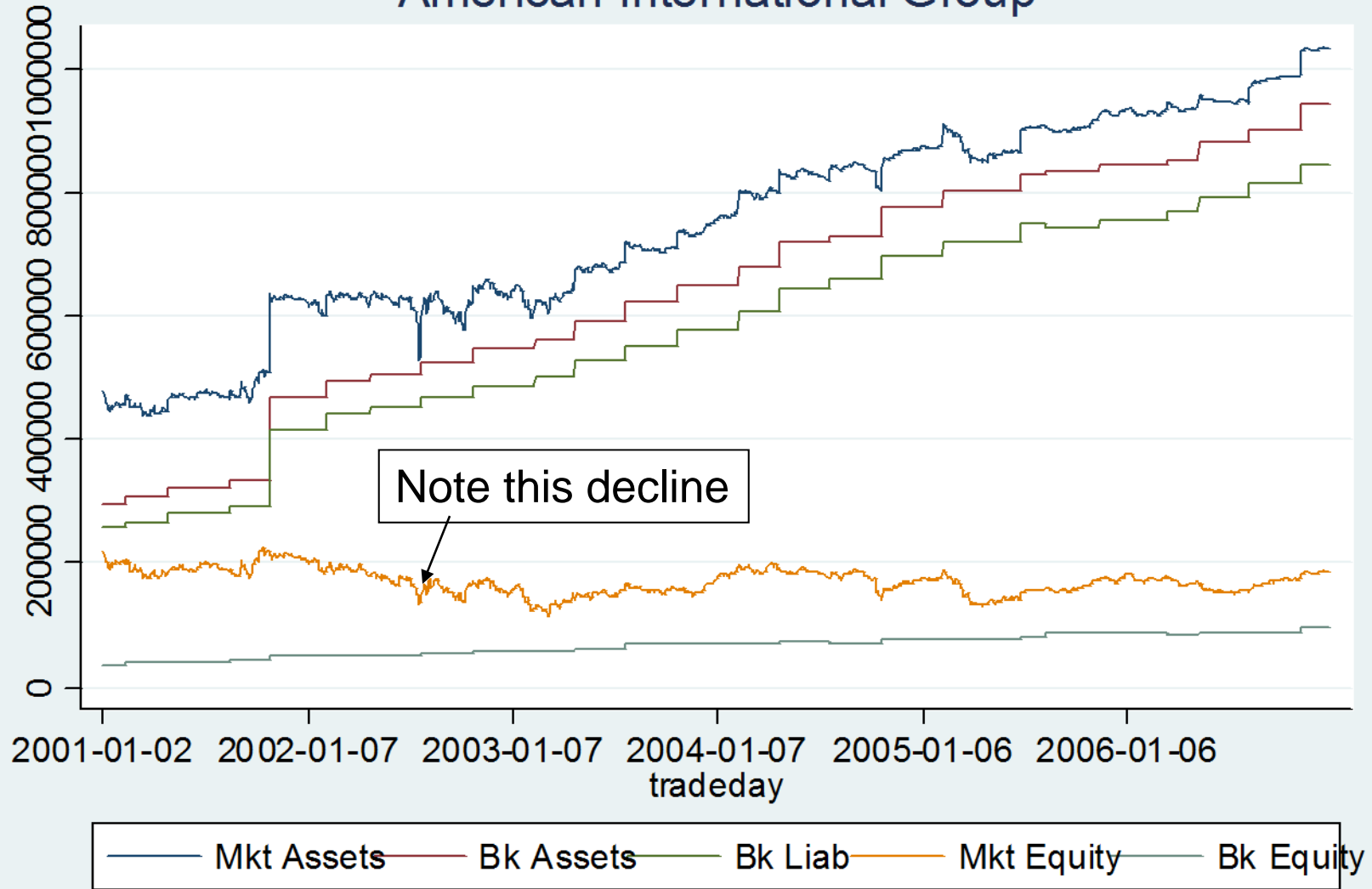
where,

$$d_1 = \frac{\ln \left( \frac{\text{Assets}}{B_0} \right) + \frac{1}{2} \sigma_{\text{Assets}}^2 T}{\sigma_{\text{Assets}} \sqrt{T}}$$

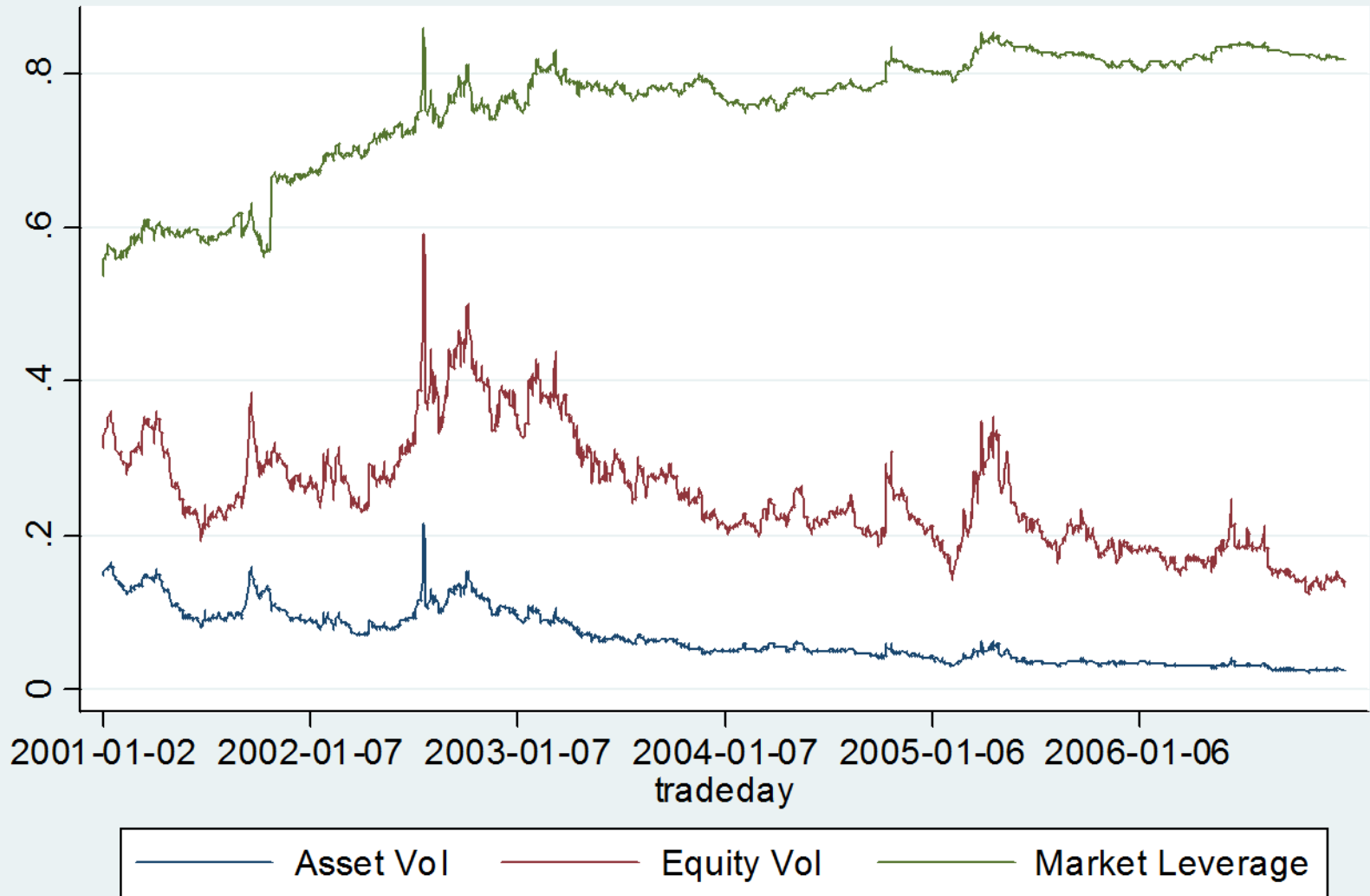
$$d_2 = d_1 - \sigma_{\text{Assets}} \sqrt{T}$$

All assets are market assets as opposed to accounting values.

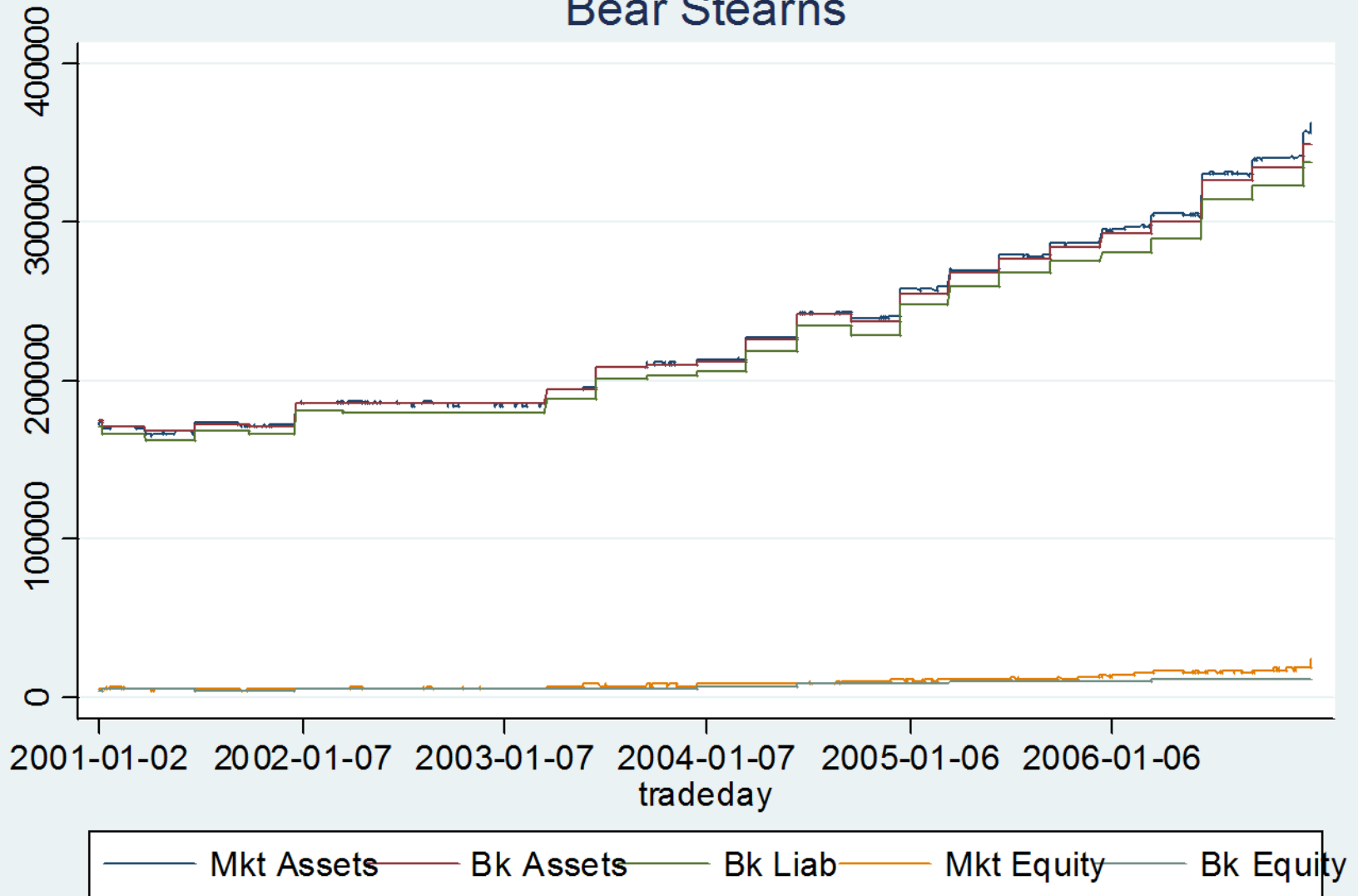
# American International Group



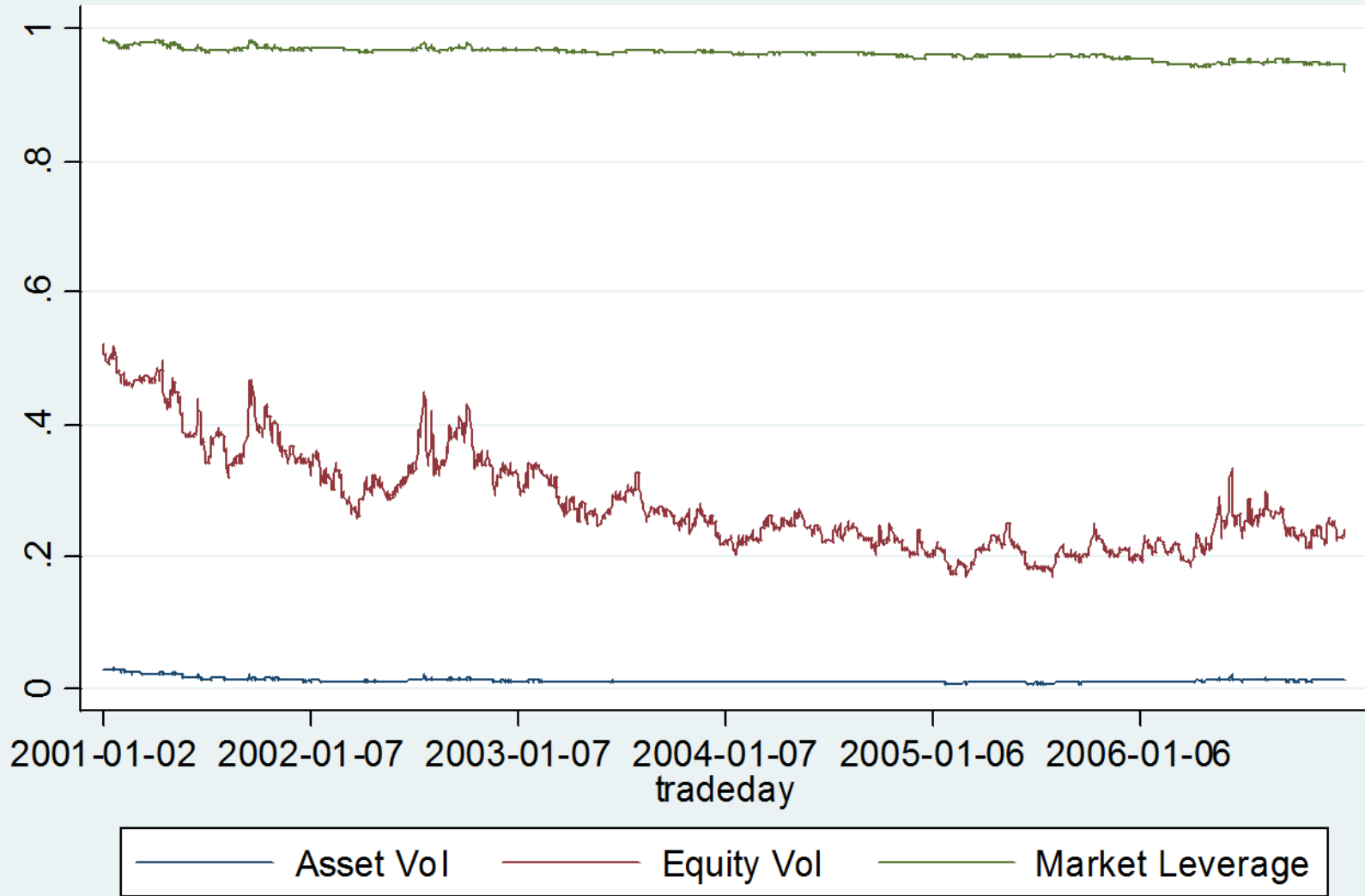
# American International Group



# Bear Stearns



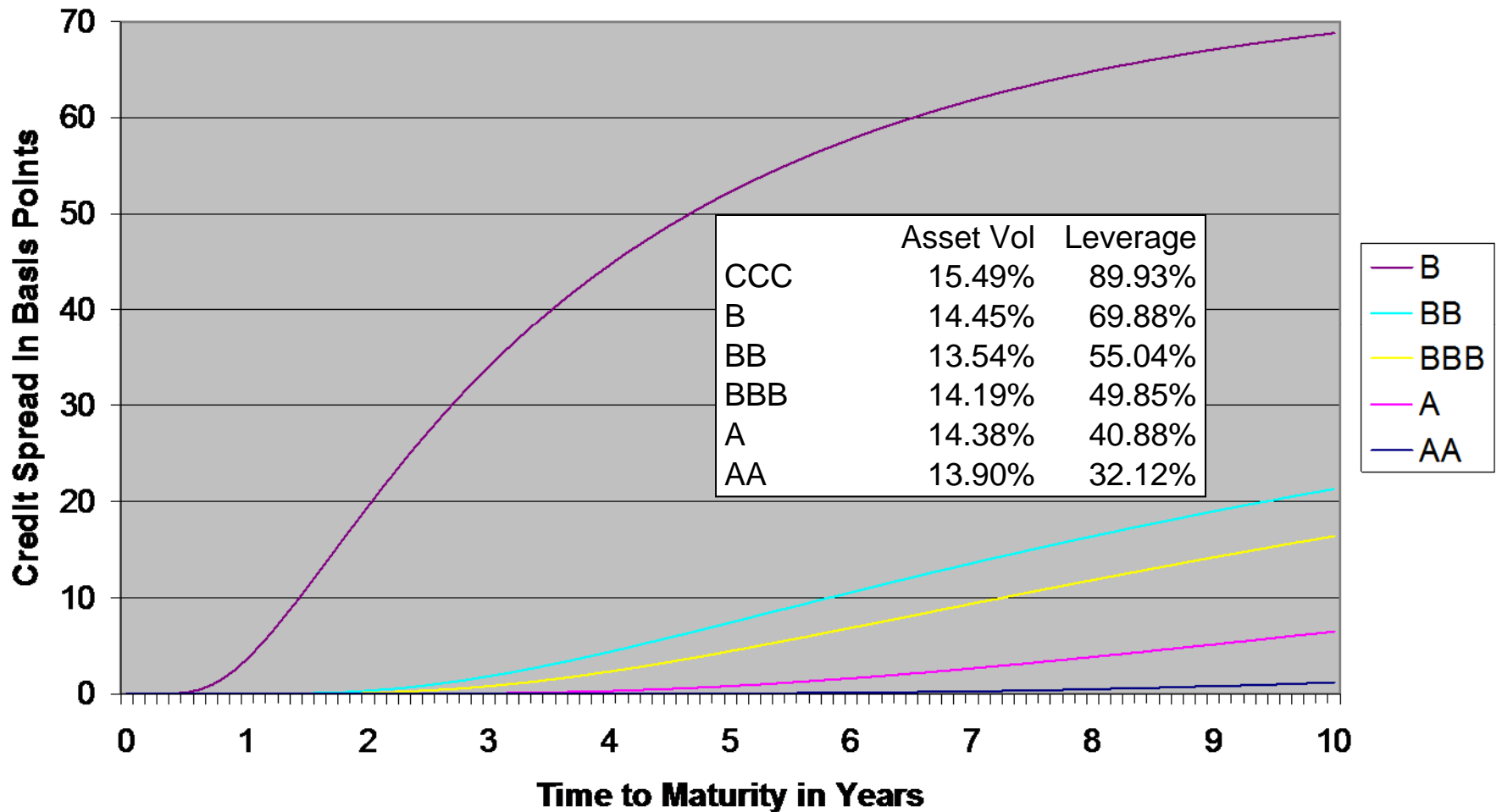
# Bear Stearns



# The Process

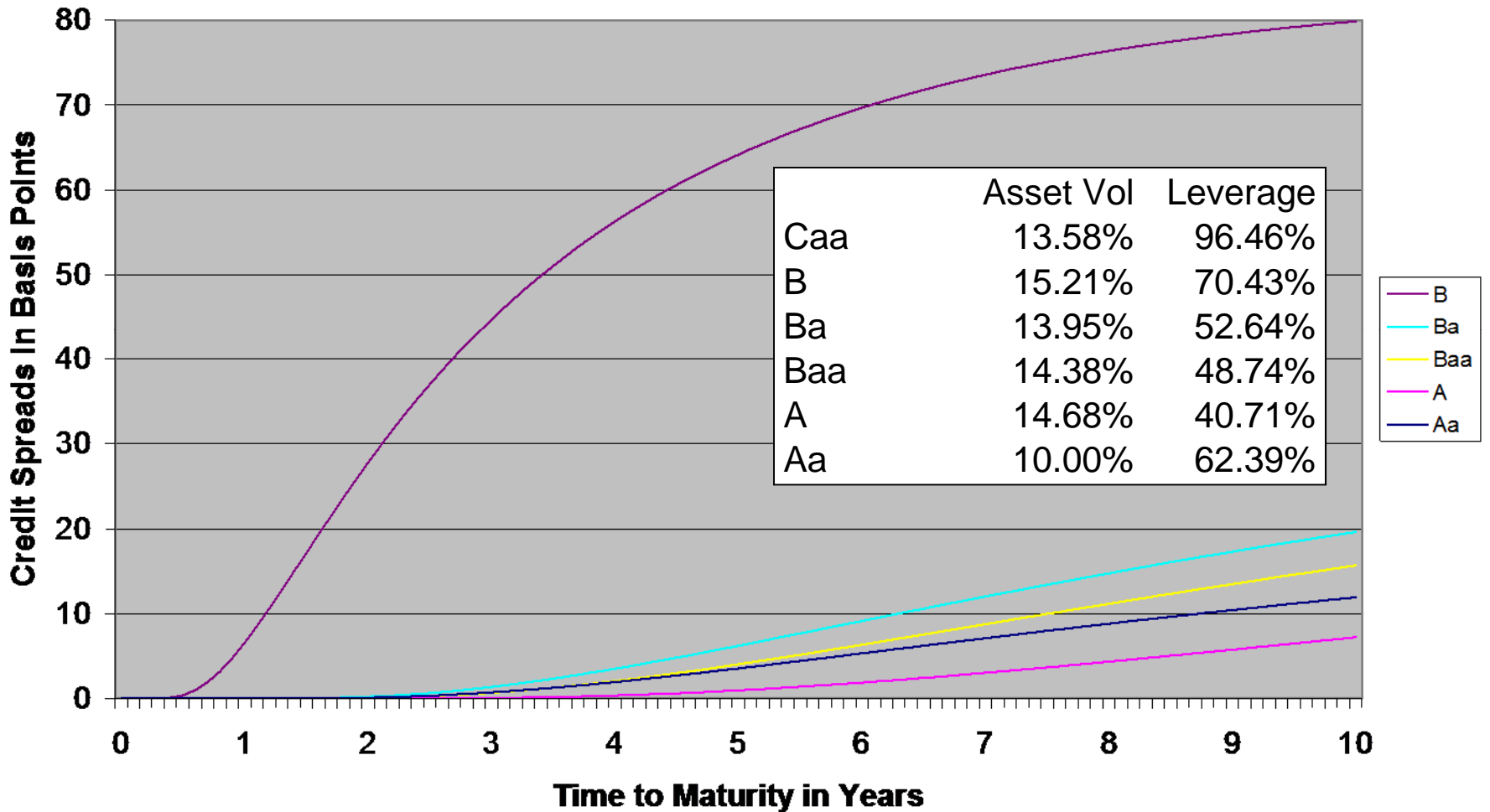
- Use Newton - Raphson to solve for asset value and volatility for each firm on each trading day substituting actual value of liabilities for discounted value (assumes liabilities grow at the risk free rate similar to assets in the model)
- Calculate Merton model spread for each of 4 tenors (one, two, five and ten years)
- Relate spreads and slopes as observed in the market to the modeled Merton spreads and Libor swap rates (risk free) and their calculated slopes

## Modified Merton Credit Spreads by S&P Rating at Median Parameter Values

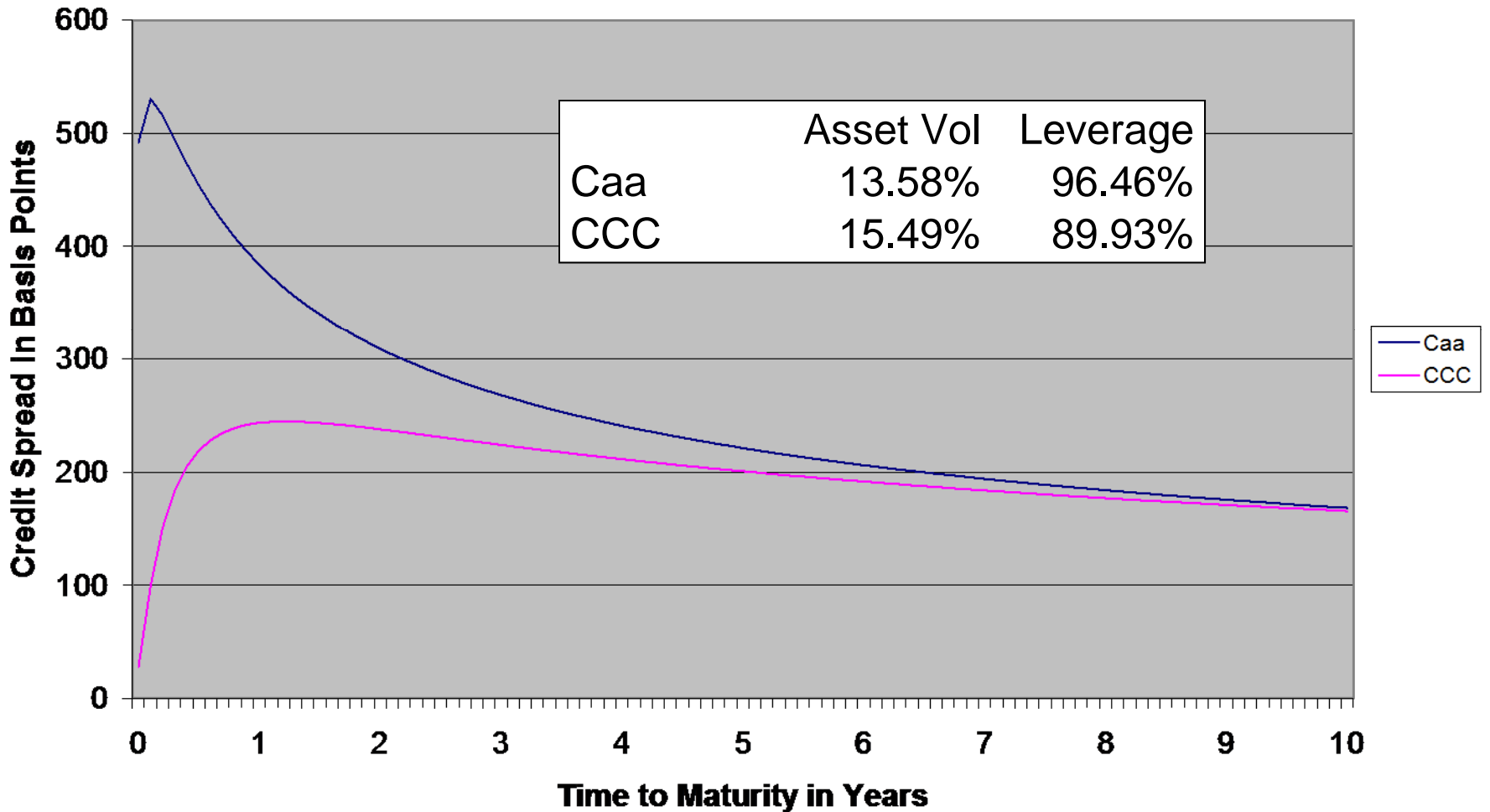




## Modified Merton Credit Spreads by Moody's Rating at Median Parameter Values

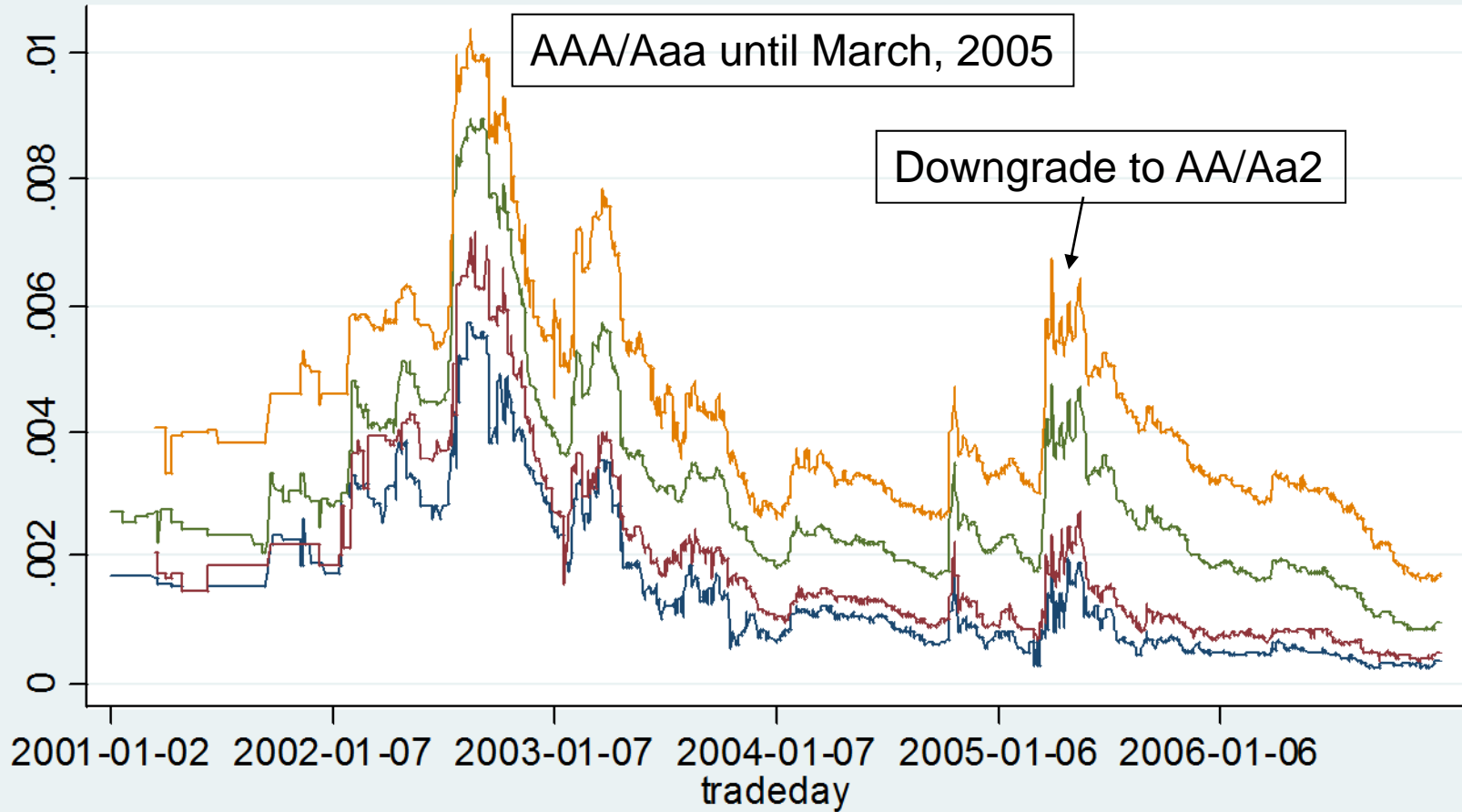


## S&P CCC/Moody's Caa Modified Merton Credit Spreads at Median Parameter Values



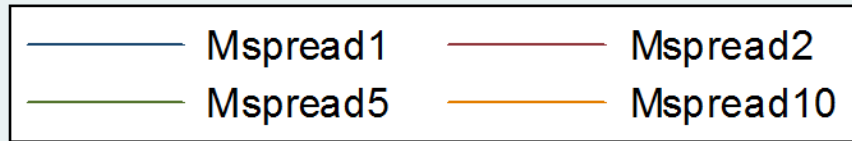
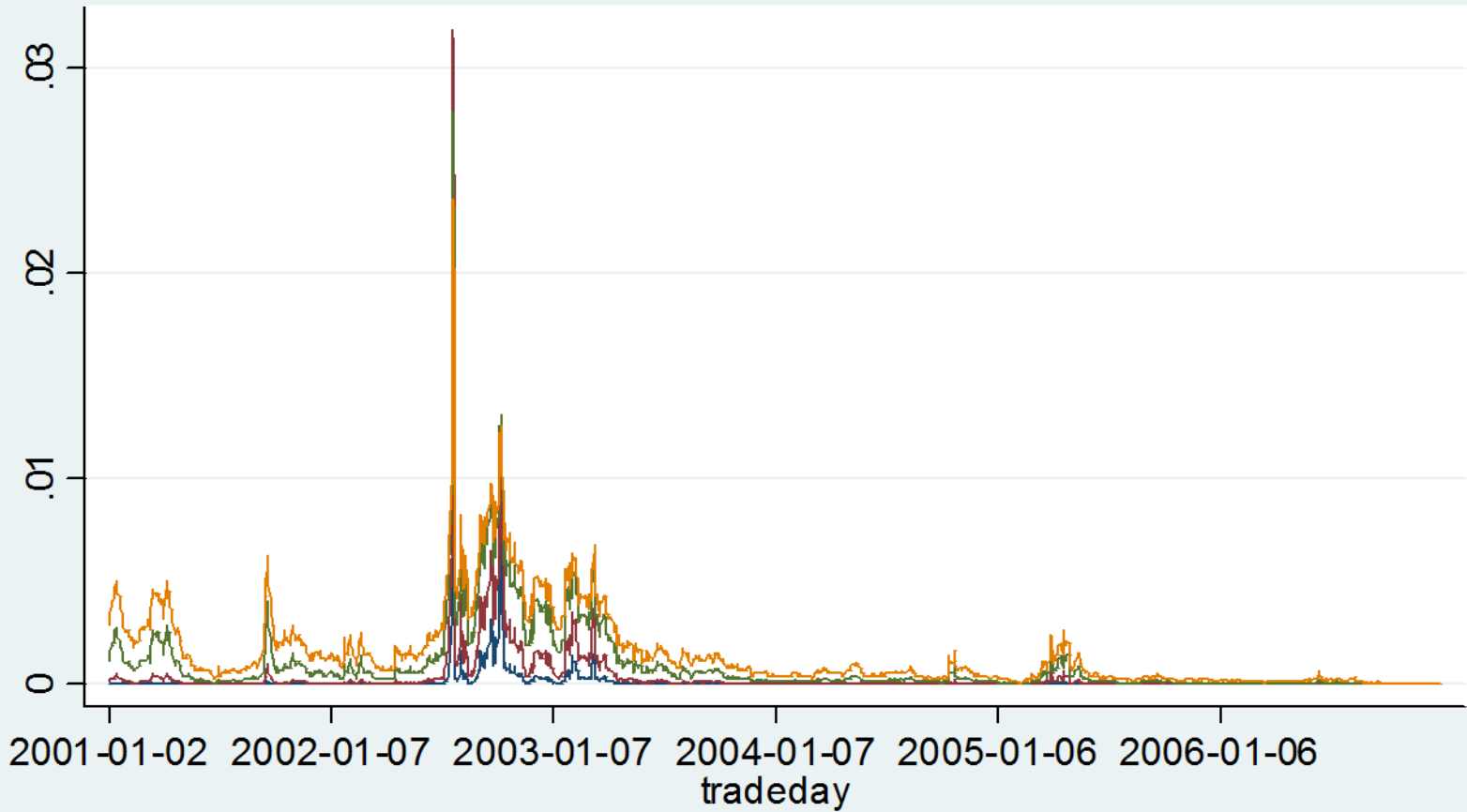
# Market Credit Spreads

## American International Group



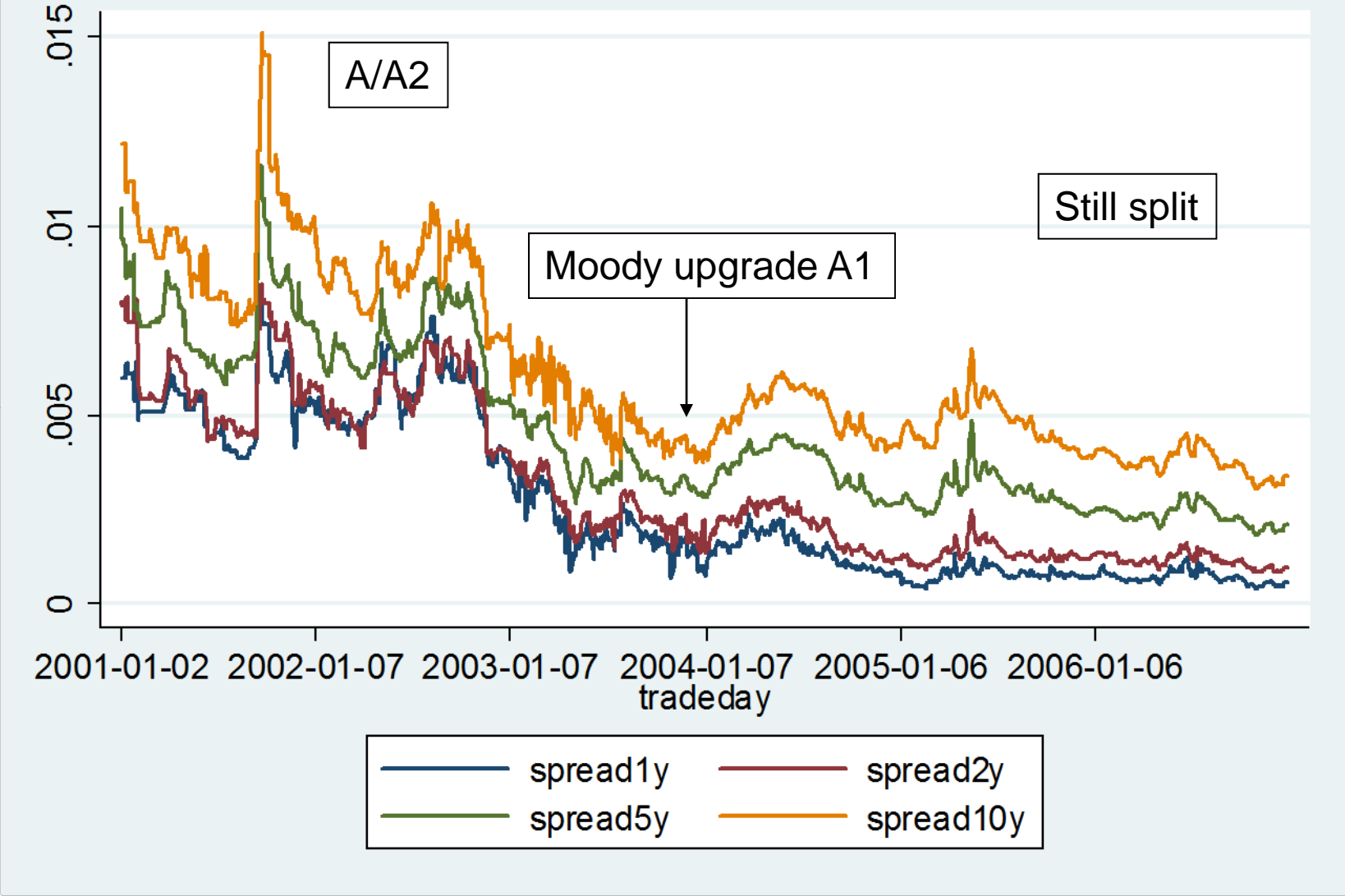
# Merton Credit Spreads

## American International Group



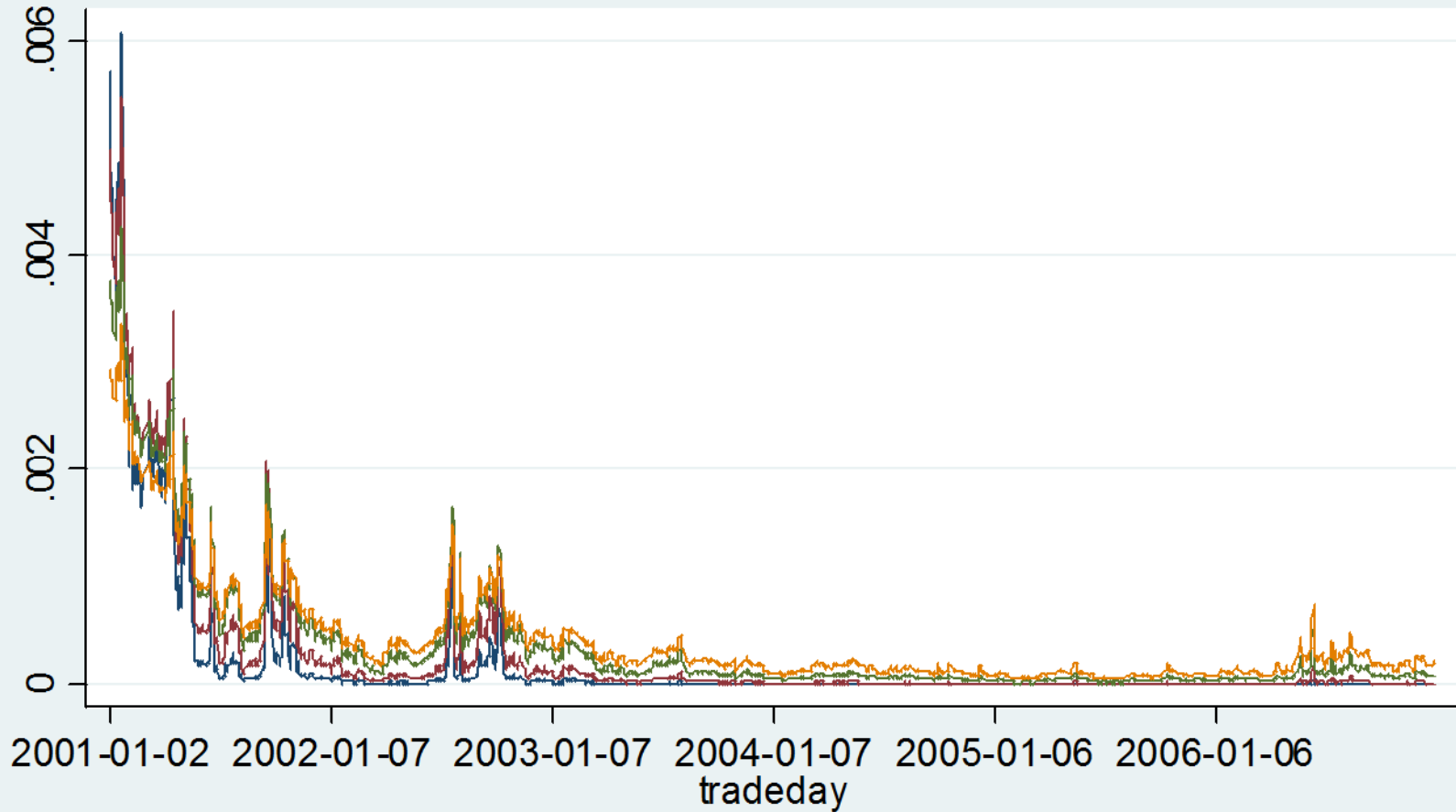
# Market Credit Spreads

## Bear Stearns



# Merton Credit Spreads

## Bear Stearns



## S&P Rat. Based Comp. of Mean Actual & Modified Merton Model Spreads in BP

Rating	5 Year Obs	One Year		Two Years		Five Years		Ten Years	
		Markit	Merton	Markit	Merton	Markit	Merton	Markit	Merton
AAA	1,131	20	2	24	5	34	16	47	26
AA+	69	18	0	25	2	41	10	59	20
AA	5,735	13	0	15	2	23	5	33	11
AA-	4,026	13	3	15	5	24	9	35	14
A+	16,834	20	2	23	5	34	11	46	18
A	27,679	25	3	29	5	42	12	56	21
A-	17,936	20	6	24	8	38	15	51	27
BBB+	23,488	47	14	50	16	69	25	83	37
BBB	29,986	74	35	78	36	100	43	113	52
BBB-	19,797	94	25	100	25	135	33	151	44
BB+	6,473	164	107	181	86	230	78	245	80
BB	7,019	159	36	177	33	235	40	257	51
BB-	4,610	148	97	167	63	239	54	265	60
B+	4,800	245	199	273	132	328	102	340	96
B	2,768	358	440	387	309	426	229	419	199
B-	1,612	266	595	330	404	400	275	420	222
CCC+	2,433	770	458	886	313	980	205	917	156
CCC	2,170	1,113	609	1,297	376	1,090	233	1,043	178
CCC-	47	1,138	2,924	1,021	1,800	962	1,047	945	741
CC	3	4,444	7,137	3,734	3,483	3,570	1,483	3,508	845
C	189	5,169	1,111	4,561	579	4,026	265	3,932	157

## Moody Rat. Based Comp. of Mean Actual & Modified Merton Model Spreads in BP

Rating	5 Year Obs	One Year		Two Years		Five Years		Ten Years	
		Markit	Merton	Markit	Merton	Markit	Merton	Markit	Merton
Aaa	1,068	19	2	24	5	34	12	47	20
Aa1	2,567	9	3	12	4	20	5	30	7
Aa2	5,134	17	1	20	4	30	12	41	19
Aa3	6,308	15	1	18	2	27	5	39	9
A1	11,164	17	3	20	5	32	10	45	17
A2	25,227	23	6	26	8	38	15	51	25
A3	17,642	26	2	29	4	44	12	57	24
Baa1	21,721	50	13	52	17	72	24	83	35
Baa2	30,430	70	35	74	35	93	43	106	53
Baa3	19,163	85	31	93	29	123	36	141	46
Ba1	8,797	93	20	100	19	151	25	160	35
Ba2	8,603	124	62	144	48	193	47	216	55
Ba3	5,754	190	94	210	67	258	61	286	65
B1	3,508	362	455	403	299	435	208	439	176
B2	3,754	158	184	177	131	278	105	298	99
B3	4,161	285	203	319	153	375	129	381	122
Caa1	1,482	686	674	812	470	901	321	878	251
Caa2	1,955	1,240	664	1,390	416	1,394	248	1,258	176
Caa3	173	3,988	2,317	3,556	1,267	3,114	633	2,947	404
Ca	194	5,045	1,083	4,435	565	3,932	259	3,839	154



# Modeling Market Observed Credit Spreads (T-Values)

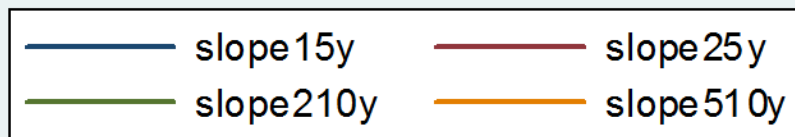
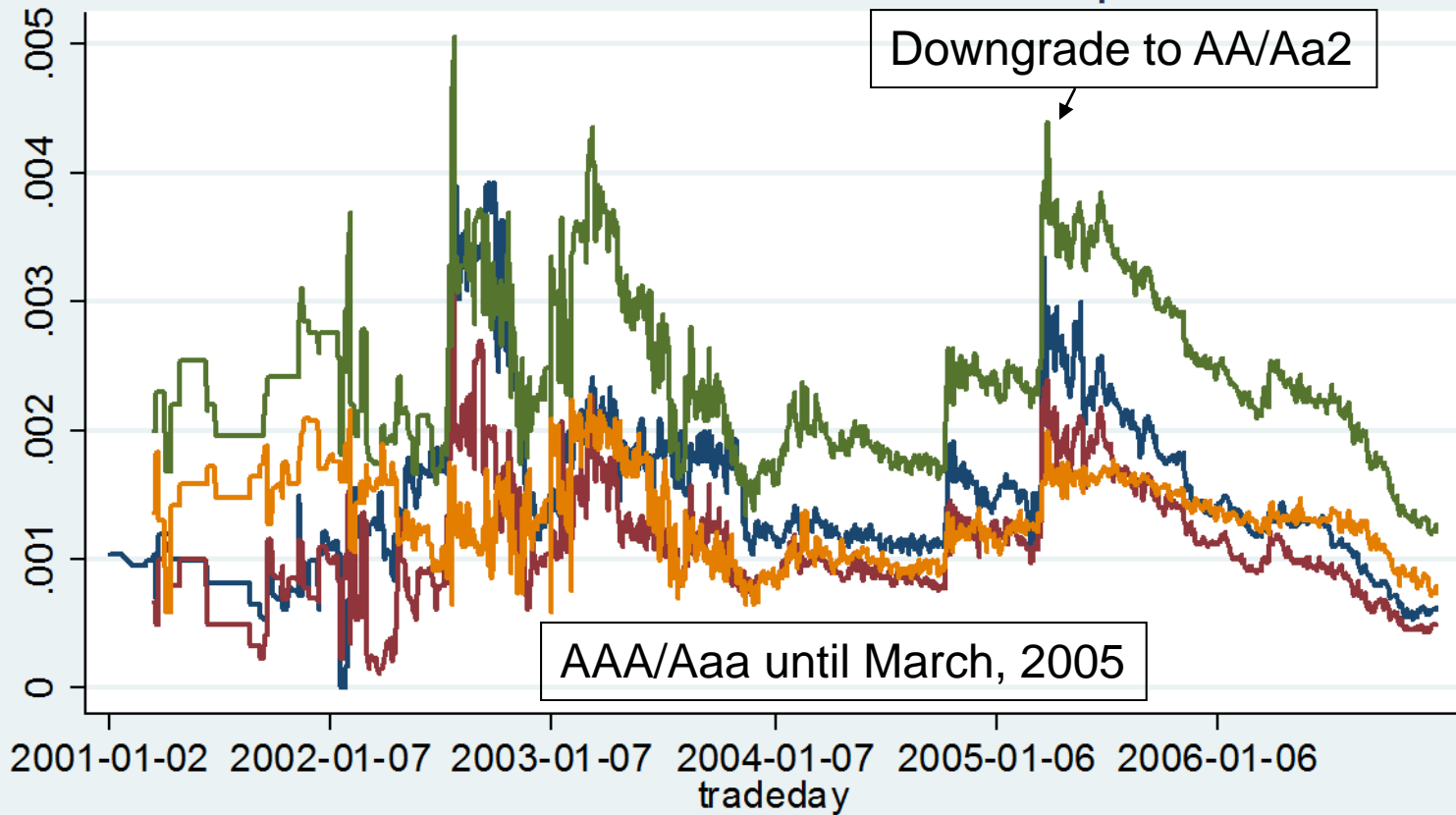
Dependent Tenor	Independent		
	Constant (BP)	Merton Spread	Swap Rate
1 Year	82*** (34.7)	29.2%*** (6.4)	
	140*** (10.1)	28.7%*** (6.2)	-17.7%*** (4.6)
2 Year	87*** (33.1)	44.9%*** (6.7)	
	153*** (10.6)	44%*** (6.6)	-18%*** (4.8)
5 Year	108*** (43.4)	54.8%*** (8.8)	
	193*** (8.2)	53.7%*** (8.9)	-19.6%*** (3.5)
10 Year	116*** (43.9)	57.5%*** (9.5)	
	236*** (3.7)	57.3%*** (9.6)	-24.3%* (1.8)

Linear Panel Data Regression (Fixed effects, Cluster firm dimension)

\*, \*\*, \*\*\* Significance at 90%, 95%, 99%

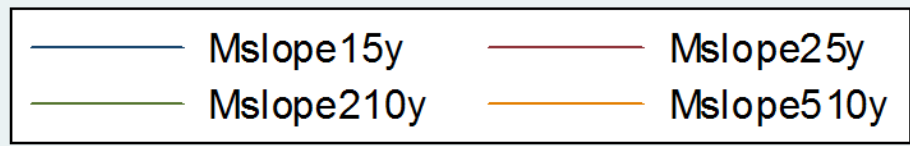
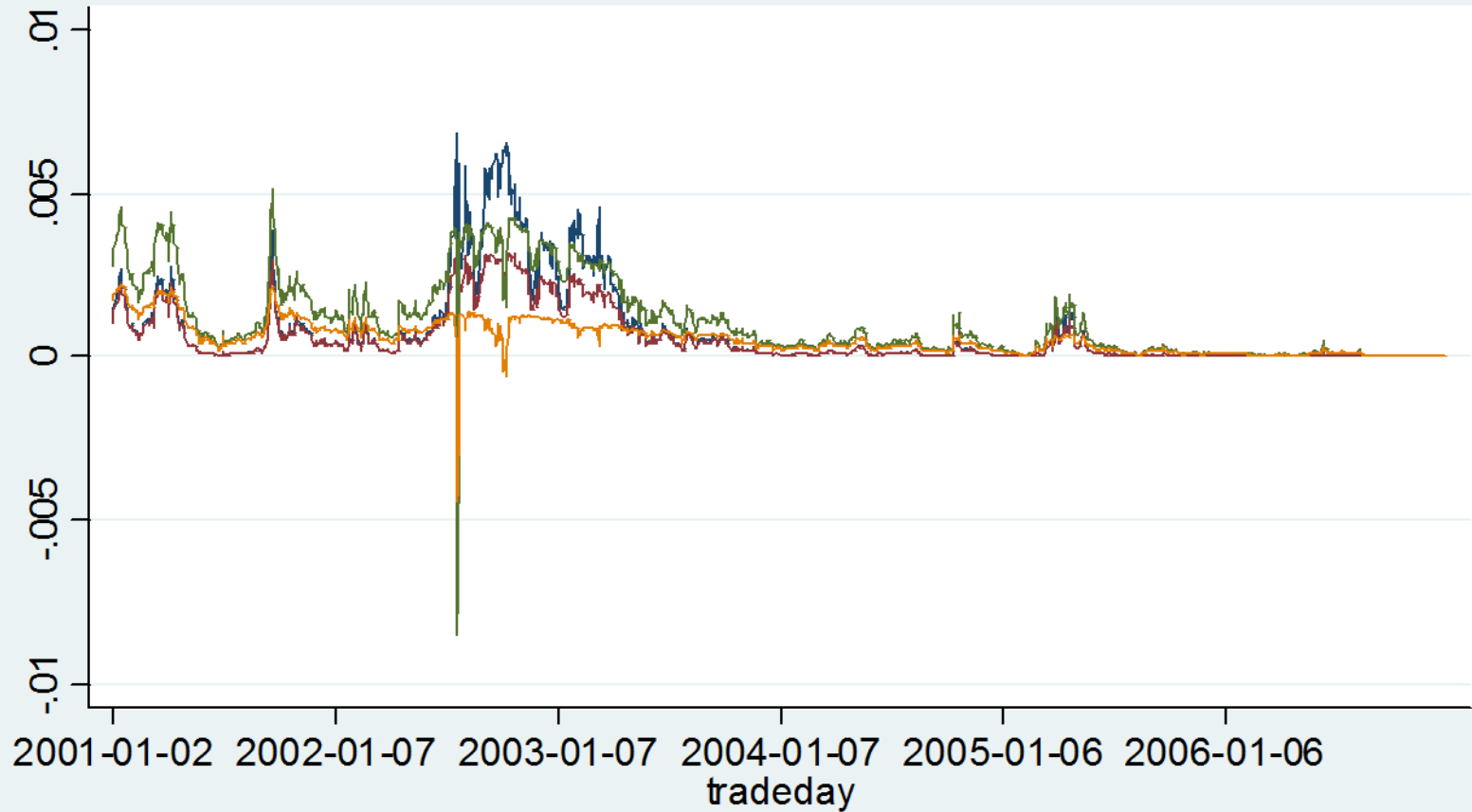
# Market Credit Spread Slopes

## American International Group



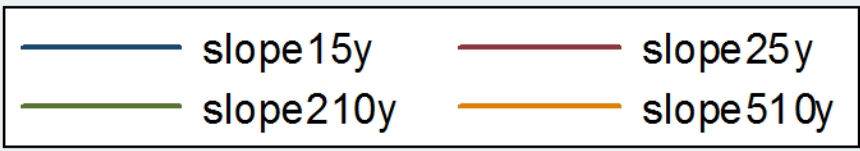
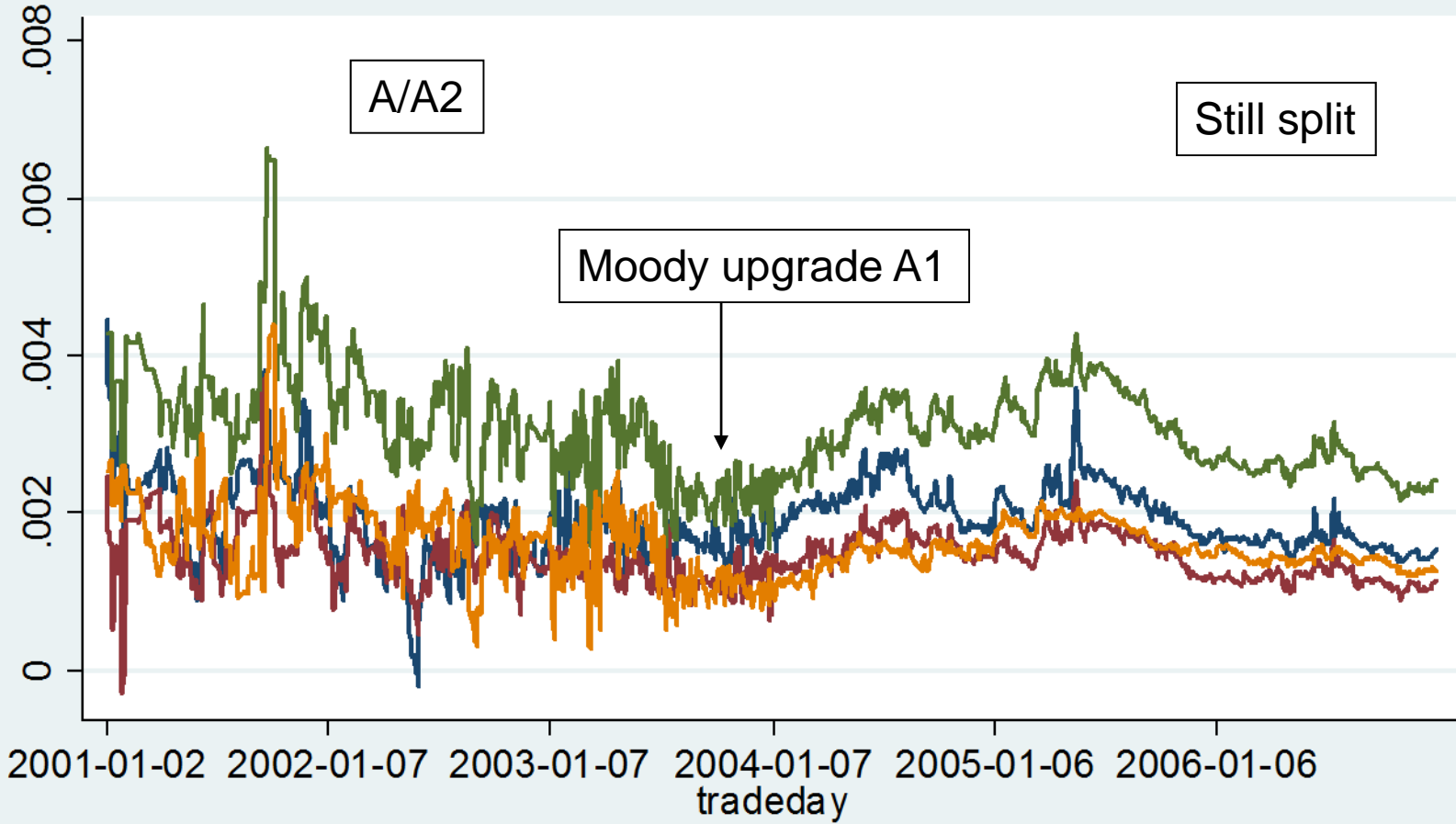
# Merton Credit Spread Slopes

## American International Group



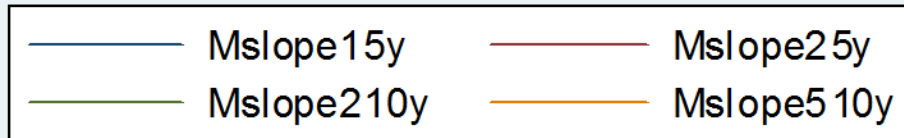
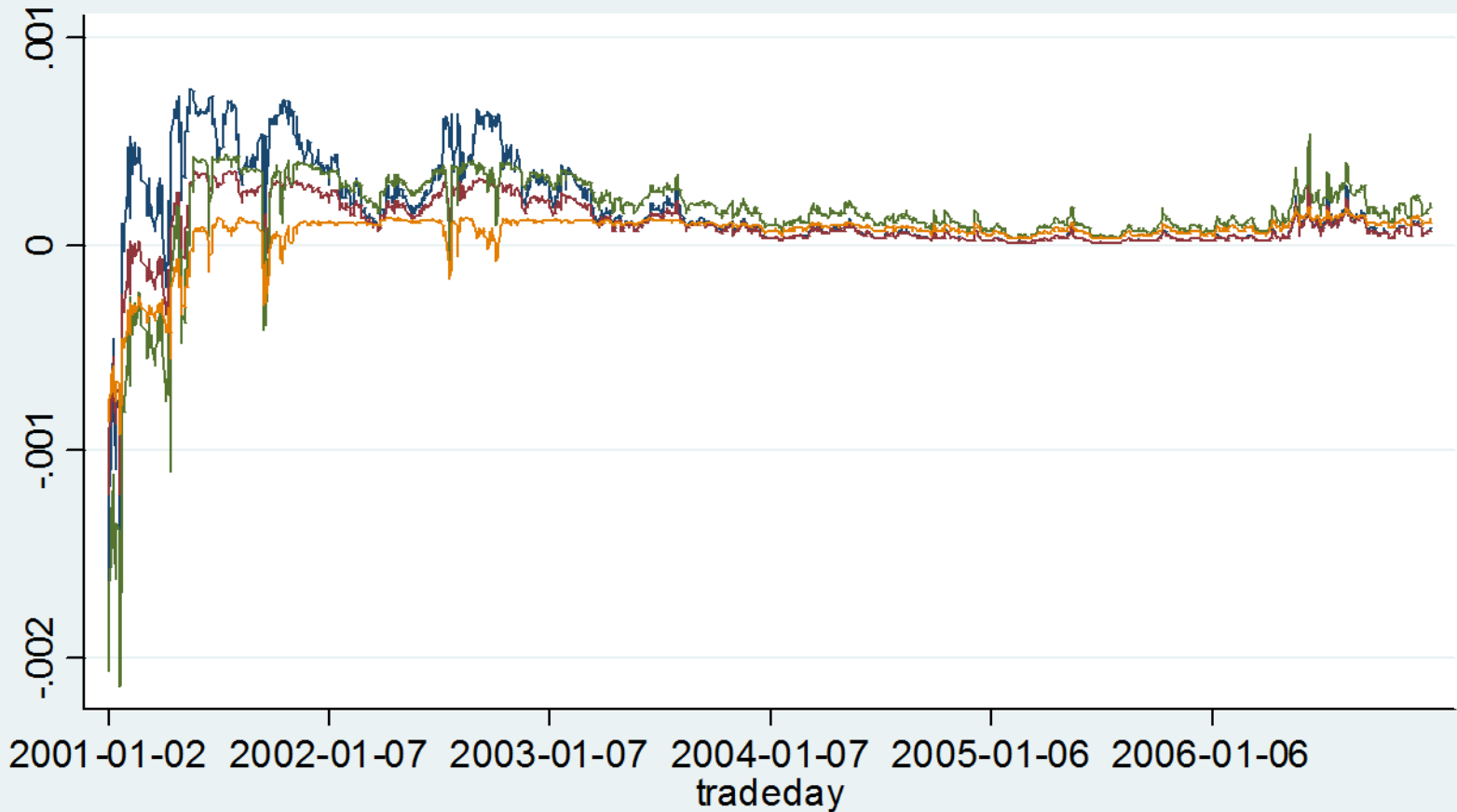
# Market Credit Spread Slopes

## Bear Stearns



# Merton Credit Spread Slopes

## Bear Stearns



## S&P Rat. Based Comp. of Mean Actual & Modified Merton Model SLOPES in BP

Rating	5 Year	Five Yr - One Yr		Five Yr - Two Yr		Ten Yr - Two Yr		Ten Yr - Five Yr	
	Obs	Markit	Merton	Markit	Merton	Markit	Merton	Markit	Merton
AAA	1,131	14	14	10	10	23	21	13	10
AA+	69	23	10	16	9	34	18	18	9
AA	5,735	11	5	8	4	18	9	9	6
AA-	4,026	11	7	9	5	20	9	11	4
A+	16,834	13	8	10	6	23	13	12	7
A	27,679	16	8	12	7	27	16	15	9
A-	17,936	17	10	12	7	27	19	14	11
BBB+	23,488	23	11	18	8	33	21	15	12
BBB	29,986	26	8	20	8	35	16	15	9
BBB-	19,797	41	8	31	8	51	19	20	11
BB+	6,473	62	(29)	44	(8)	67	(6)	22	2
BB	7,019	79	4	58	7	82	18	21	11
BB-	4,610	85	(43)	63	(9)	96	(3)	32	6
B+	4,800	83	(97)	60	(29)	84	(36)	21	(6)
B	2,768	65	(212)	33	(80)	40	(109)	5	(30)
B-	1,612	137	(320)	76	(129)	97	(182)	20	(53)
CCC+	2,433	209	(254)	104	(108)	48	(157)	(61)	(49)
CCC	2,170	16	(376)	(31)	(143)	(138)	(197)	(90)	(55)
CCC-	47	(176)	(1,877)	(59)	(753)	(76)	(1,059)	(17)	(306)
CC	3	(873)	(5,654)	(164)	(2,001)	102	(2,639)	(179)	(638)
C	189	(1,151)	(846)	(535)	(314)	(618)	(423)	(86)	(108)

## Moody Rat. Based Comp. of Mean Actual & Modified Merton Model SLOPES in BP

Rating	5 Year Obs	Five Yr - One Yr		Five Yr - Two Yr		Ten Yr - Two Yr		Ten Yr - Five Yr	
		Markit	Merton	Markit	Merton	Markit	Merton	Markit	Merton
Aaa	1,068	15	10	10	8	23	15	13	8
Aa1	2,567	10	2	7	2	17	4	10	2
Aa2	5,134	13	10	10	8	21	15	11	8
Aa3	6,308	12	4	9	3	21	7	12	4
A1	11,164	15	7	12	5	25	12	13	7
A2	25,227	15	10	11	7	25	16	14	9
A3	17,642	17	10	13	8	28	21	15	12
Baa1	21,721	21	11	16	8	30	18	14	10
Baa2	30,430	23	8	18	8	32	18	14	10
Baa3	19,163	38	6	28	7	48	17	19	10
Ba1	8,797	52	4	39	6	65	16	23	10
Ba2	8,603	68	(15)	49	(1)	73	6	23	7
Ba3	5,754	71	(33)	51	(7)	79	(2)	25	4
B1	3,508	73	(247)	46	(90)	61	(123)	12	(32)
B2	3,754	120	(80)	90	(26)	120	(32)	27	(6)
B3	4,161	85	(74)	52	(24)	68	(31)	14	(7)
Caa1	1,482	217	(352)	121	(149)	89	(219)	(30)	(70)
Caa2	1,955	177	(416)	44	(168)	(99)	(240)	(141)	(72)
Caa3	173	(873)	(1,684)	(441)	(634)	(616)	(863)	(167)	(229)
Ca	194	(1,121)	(823)	(521)	(305)	(602)	(412)	(84)	(106)

# Modeling Market Observed Credit Spread Slopes (T-Values)

Dependent Tenor	Independent		
	Constant (BP)	Merton Slope	Swap Slope
5 Year - 1 Year	33*** (77.3)	11.6%*** (3.4)	
	46*** (14.7)	11.5%*** (3.4)	-12.0%*** (4.0)
5 Year - 2 Year	22*** (378.8)	19.5%*** (4.0)	
	34*** (17.2)	19.5%*** (4.1)	-16.7%*** (6.0)
10 Year - 2 Year	35*** (192.2)	19.8%*** (4.3)	
	58*** (21.1)	19.8%*** (4.5)	-18.1%*** (8.2)
10 Year - 5 Year	12*** (42.8)	18.9%*** (3.6)	
	24*** (25.5)	18.9%*** (3.8)	-20.2%*** (11.4)

Linear Panel Data Regression (Fixed effects, Cluster firm dimension)

\*, \*\*, \*\*\* Significance at 90%, 95%, 99%



# Conclusions

- This study uses daily CDS spreads and Libor Swap rates for risk free and finds that modeled behavior of Merton spreads is consistent with past research in that
  - Short tenor high quality firms have very low Merton spreads (introduce jumps)
  - Merton spreads provide high statistical explanatory power for observed spreads with a constant that could proxy, in part, for liquidity
  - Merton spreads are extremely sensitive to high leverage levels
  - Risk free rate levels are negatively related to spread levels
- This study utilizes CDS and Swap rate data to model the term structure of credit spreads and finds
  - Slope measures (combinations between 1,2,5,10 year points) proxied by Merton model slopes and Swap rate slopes are explained with high statistical significance
  - Merton slopes are negative at much higher credit quality levels than observed in the market data (introduce stationary leverage ratios)
  - Credit spread slopes are negatively related to risk free slope levels