

# What can Garch(1,1) Modeling Tell Us About the Degree of Overreaction in Single Stock Option Volatility?

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December 7<sup>th</sup>, 2005

# Introduction

- Mean reversion (Poterba and Summers, 1986) or Overreaction (Stein, 1989) in option market volatility is well documented
- Can the degree of overreaction be measured and trading rules applied?
  - Focus upon fitting another mean reverting process – Garch (1,1) to the underlier – and work with changes in Excess Implied Volatility (Garleanu, Pederson & Poteshman, 2005)
  - Define Excess as Implied vol – Garch vol
  - Look at constant maturity, at-the-money implied volatility for 30 and 91 days to maturity

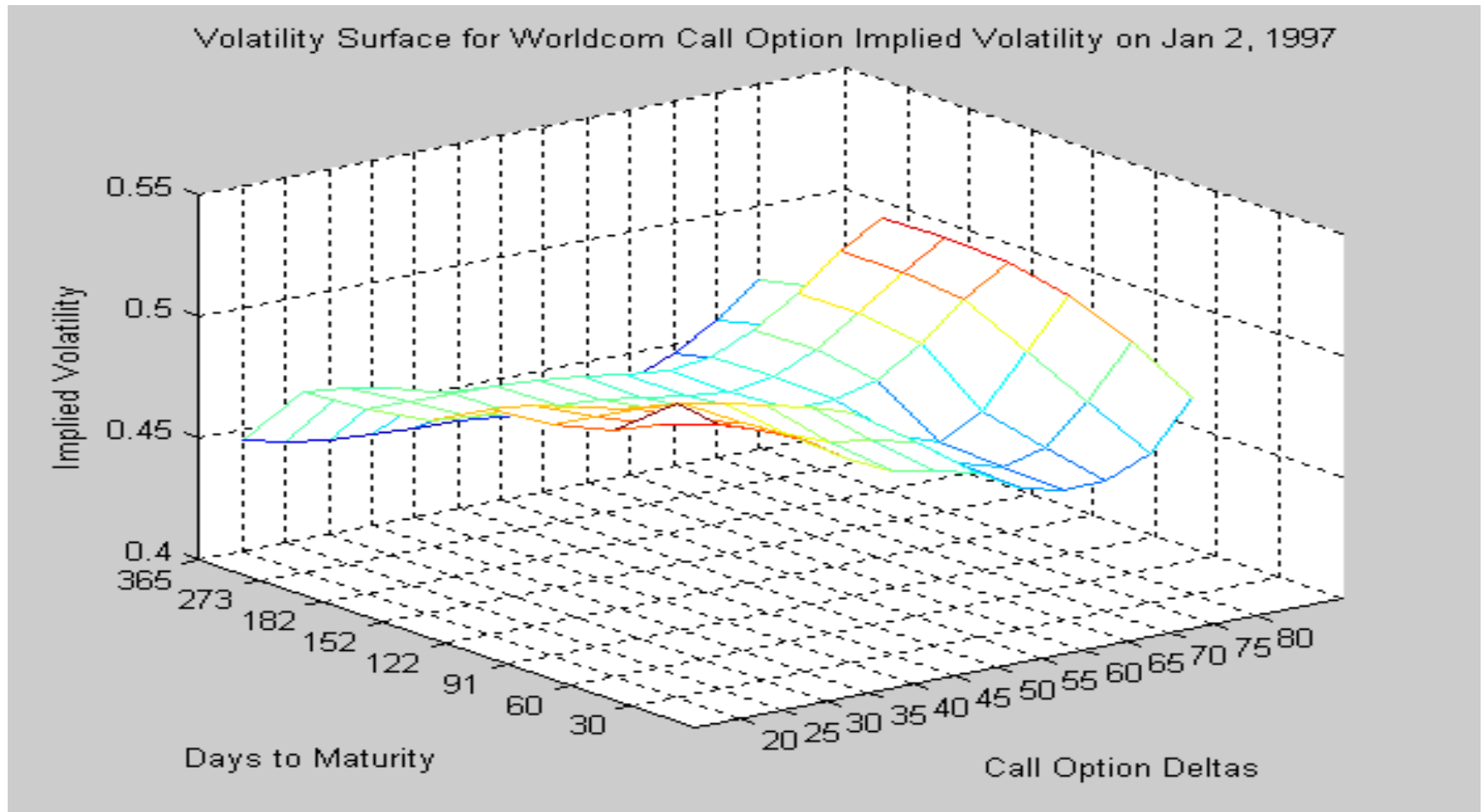
# Data

- Daily standardized constant maturity, at-the-money implied volatility and return data for single name stocks available beginning in 1996 from OptionMetrics
  - Examine 1997 implied volatilities fitting a rolling 1 year Garch (1,1)
  - Examine 1998 implied volatilities fitting a rolling 1 year Garch (1,1)
  - Examine 1998 implied volatilities fitting a rolling 2 year Garch (1,1)

# Option Metrics Standardized Implied Vols (1)

- Collect all option trades each day for a firm
- Determine each option implied volatility, delta and vega from binomial model, collect time to maturity and identify if put or call, converting put deltas to call equivalent deltas
- Create 2 surfaces (call and put) via kernel smoothing by time to maturity and call equivalent delta weighting each underlying option by its vega
- Simultaneously solve for strike and option premium for each point on the surface through iteration

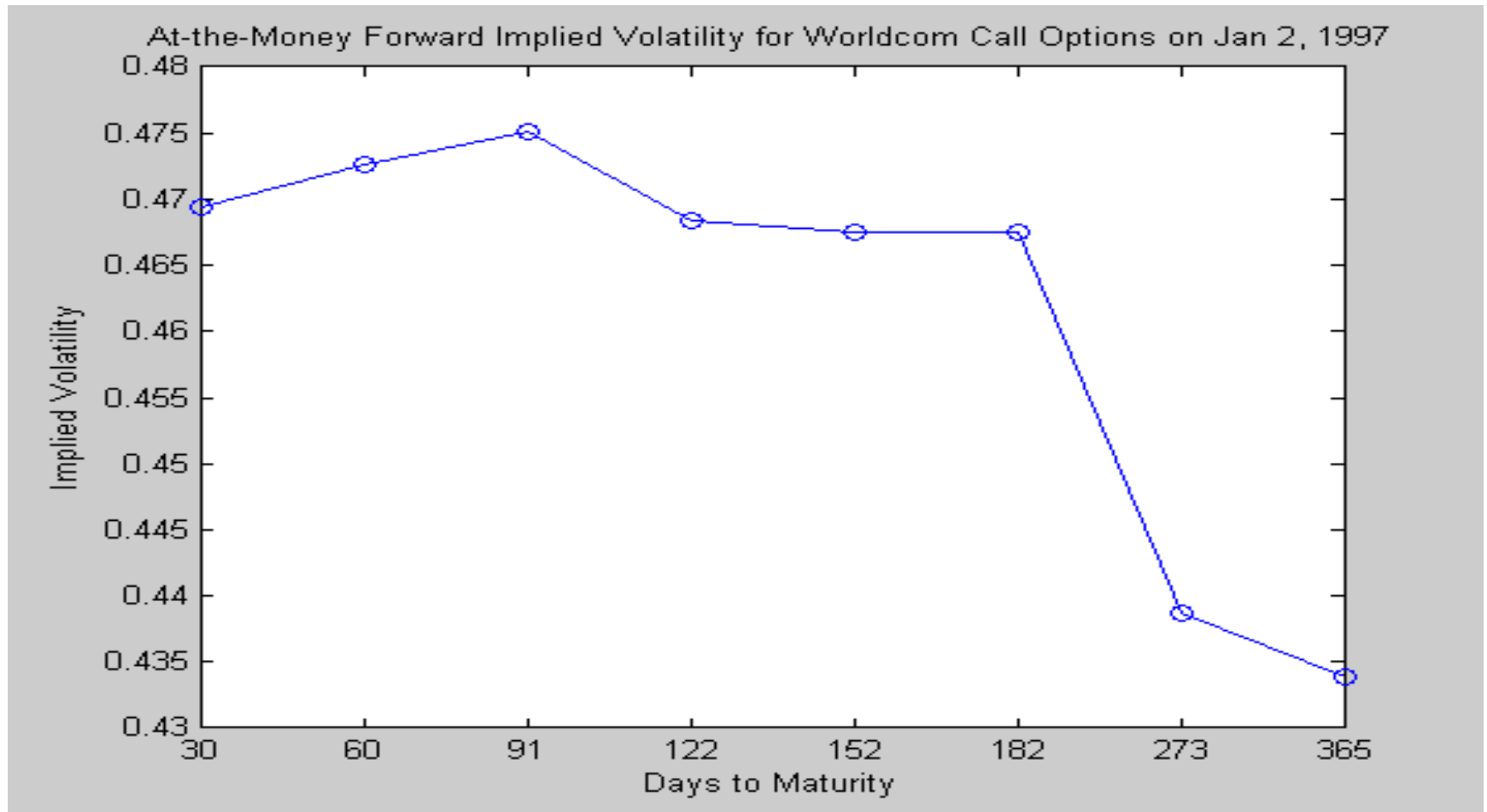
# OptionMetrics Standardized Implied Vols (2)



# OptionMetrics Standardized Implied Vols (3)

- With Vol Surface in place, determine forward price of underlier for desired term to maturity and option type (call or put)
- Linearly interpolate over surface strike prices and time to maturity for at the money forward implied volatility
- Repeat for varying time to maturity

# OptionMetrics Standardized Implied Vols (4)



# Potential Issues with the Data

- Gathering extreme implied volatility changes may include data errors of two types
  - Genuine information driven changes to option price between stock market close at 4PM EST and option market close at 4:02PM EST
    - Bakshi, Cao, and Chen (2000) somewhat lessens this concern finding that intraday or interday call and underlier prices sometimes move in opposite directions
  - Stale stock pricing data carried to 4PM
  - Both will lead to variation in reported implied volatility lacking economic meaning



# Controlling for Data Error

- Difficult
  - Tick data examination required for option price change between 4:00 and 4:02
  - Require at least 50 daily trades in all options of a given underlier for inclusion of standardized vols in data
  - In addition to 30 and 91 day to maturity points, create slope variable (91 day minus 30 day implied vol)
    - This will at least reduce any error to a secondary time to maturity difference

# Resulting Implied Volatility Data

- One trade day observation is one with both a 30 and 91 day to maturity standardized ATM Volatility

1997			1998		
<i>Trade Day Obs</i>	<i>Firms</i>	<i>Cumulative %</i>	<i>Trade Day Obs</i>	<i>Firms</i>	<i>Cumulative %</i>
1-25	746	29.94%	1-25	855	30.92%
26-50	370	44.78%	26-50	464	47.70%
51-75	262	55.30%	51-75	290	58.19%
76-100	234	64.69%	76-100	239	66.84%
101-125	180	71.91%	101-125	169	72.95%
126-150	130	77.13%	126-150	132	77.72%
151-175	103	81.26%	151-175	108	81.63%
176-200	97	85.15%	176-200	110	85.61%
201-225	106	89.41%	201-225	107	89.48%
226-252	264	100.00%	226-253	291	100.00%

**2492**

**2765**

# The Garch (1,1) Model

t is trading days  
ε must be mean zero  
model residuals  
α is decay rate of  
return residuals

$$\sigma_t^2 = \kappa + \alpha\sigma_{t-1}^2 + \beta\varepsilon_{t-1}$$

$$\kappa > 0,$$

$$\alpha, \beta \geq 0$$

Stationary variance  
Requirement

$$\alpha + \beta < 1$$

Long run variance from  
Process - will revert to  
this value

$$\sigma^2 = \kappa / (1 - \alpha - \beta)$$

# Choosing Garch(1,1)

- Garch (1,1) is symmetric in effect on variance from directional moves in return in contradiction to Black's (1976) leverage effect (volatility goes higher on down moves lower on up moves)
- To capture negative correlation between innovations in return and volatility of return one can move to asymmetric Garch models such as AGarch, EGarch, or TGarch.
- Hagerud (1997) in assessing 45 Nordic Stocks finds Heteroskedasticity in 32, 12 of which display asymmetric behavior
  - Robust test methods for asymmetry begin with a Garch (1,1)

# Choosing Garch(1,1) cont.

- Dennis, Mayhew and Stivers (2004) find negative return-volatility correlation much more pronounced for index returns (-0.679) than individual stocks (-0.165)
  - Further, in examining 50 stocks and specifically attempting to detect asymmetric volatility with a dummy on negative returns, only 7 are statistically significant and the coefficients show a firm size bias from zero for the smallest quintile to 0.045 for the largest

# The Conditional Mean

- Two typical choices for returns are mean deviations and AR(1):

$$r_t = \bar{r} + \varepsilon_t$$

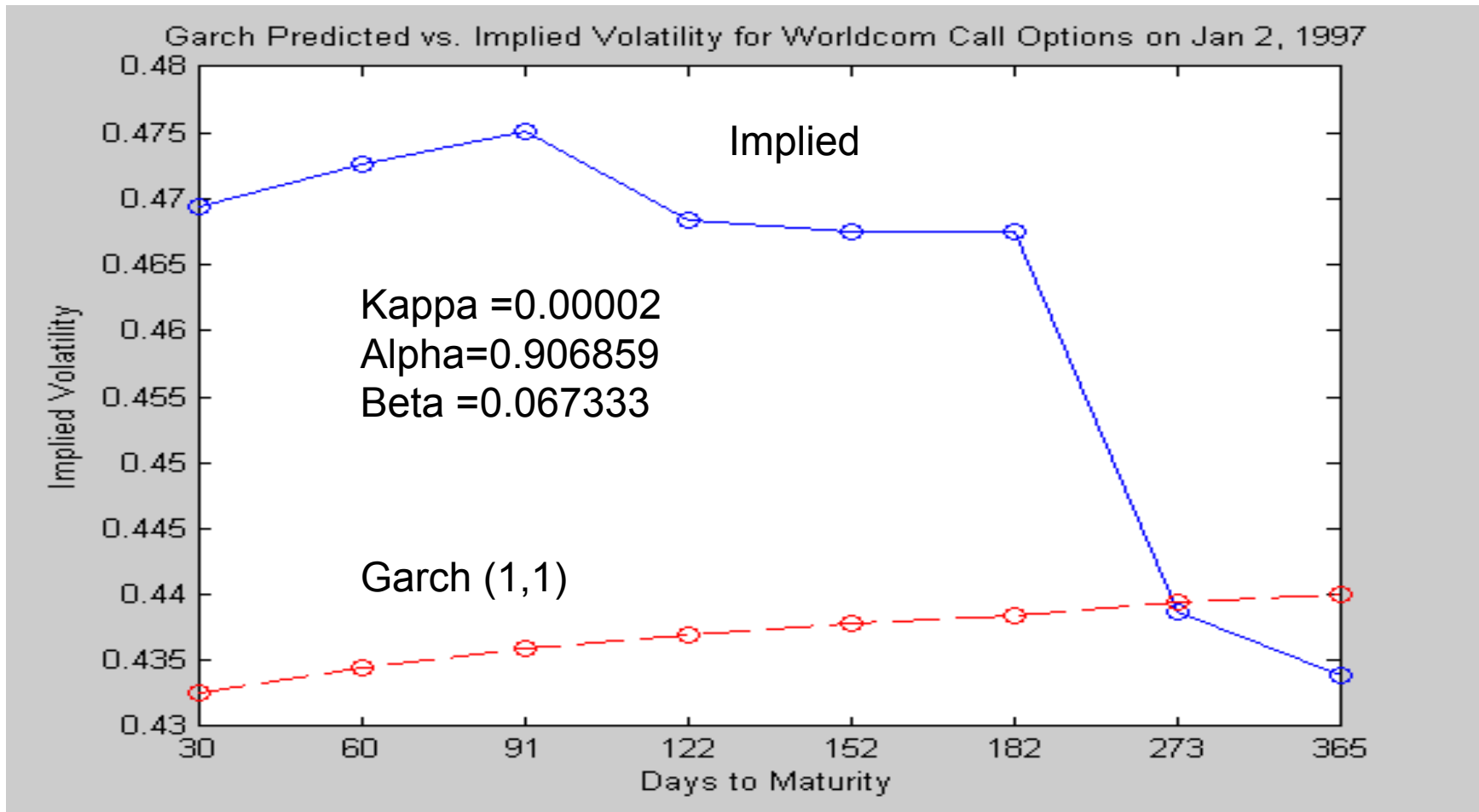
$$r_t = \alpha r_{t-1} + \varepsilon_t$$

- Alexander(2001) uses simplicity as a reason for mean deviations in most models. Hagerud (1997) finds 12 of 45 Nordic firms show evidence of return autocorrelation.
- Note that the symmetry in Garch(1,1) permits separate estimation of conditional mean and variance equations.

# Garch (1,1) Procedure

- Produce 1 or 2 years of mean deviations in daily returns (Hagerud, 1997, uses 5 years while Alexander, 2001, recommends several stating 1 year is the absolute minimum)
- Use these residuals to fit Kappa, Alpha and Beta
- Predict future conditional one day variances for trading day equivalent of 30 and 91 calendar days
- Sum these daily variances for average variance level over trading day equivalent of 30 or 91 days
- Convert to volatility and annualize
- Repeat each day on rolling series of returns

# Garch (1,1) Procedure Example





# The Fitting Process: Practical Consideration

- Used Matlab functions for univariate Garch fit adapted from Hamilton (1994)
  - MLE does not always converge
  - High values of alpha (decay factor) near 1 lead to drastic increases in predicted volatility while beta values near 1 are not as clear cut
  - Eliminated any record producing estimated alpha or beta  $> 0.99$
- Time consuming!
  - One year of returns took 18 hours assuming the identity of the non-converging firms were known in advance
  - Two years of returns took three days

# Garch Fitting Stats

	1997	1998
Total Firms	2492	2765
Did not converge	23	18
Data missing or bad fit	80	98

Fitted Firms                      2389    2649

December 7th, 2005

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Non-Converging Firms	
1997	1998
AMOCO CORP	AMERICAN INTL GROUP INC
ASARCO INC	ANHEUSER BUSCH COS INC
CAVALIER HOMES INC	APACHE CORP
CONTINENTAL AIRLINES INC -CL B	ASTORIA FINL CORP
CORRECTIONS CORP OF AMERICA	CHASE MANHATTAN CORP
DELTA AIR LINES INC DEL	CITICORP
DOMINION RESOURCES INC	EDWARDS AG INC
DOW CHEM CO	FORE SYSTEMS INC
FAMILY DLR STORES INC	GILLETTE CO
FEDERAL HOME LN MTG CORP	KELLOGG CO
GATX CORP	KOHL'S CORP
GENERAL ELEC CO	RAYCHEM CORP
GENERAL MTRS CORP	SONY CORP
ILLINOIS TOOL WKS INC	SUMMIT BANCORP
MARRIOTT INTL INC	TELEPORT COMM GRP -CL A
MEAD CORP	TIME WARNER INC
MERRILL LYNCH & CO INC	UNUM CORP
MOBIL CORP	WELLS FARGO & CO
PNC BANK CORP	
SBC COMMUNICATIONS INC	
SNAP ON INC	
WEYERHAEUSER CO	
YPF SOCIEDAD ANONIMA	

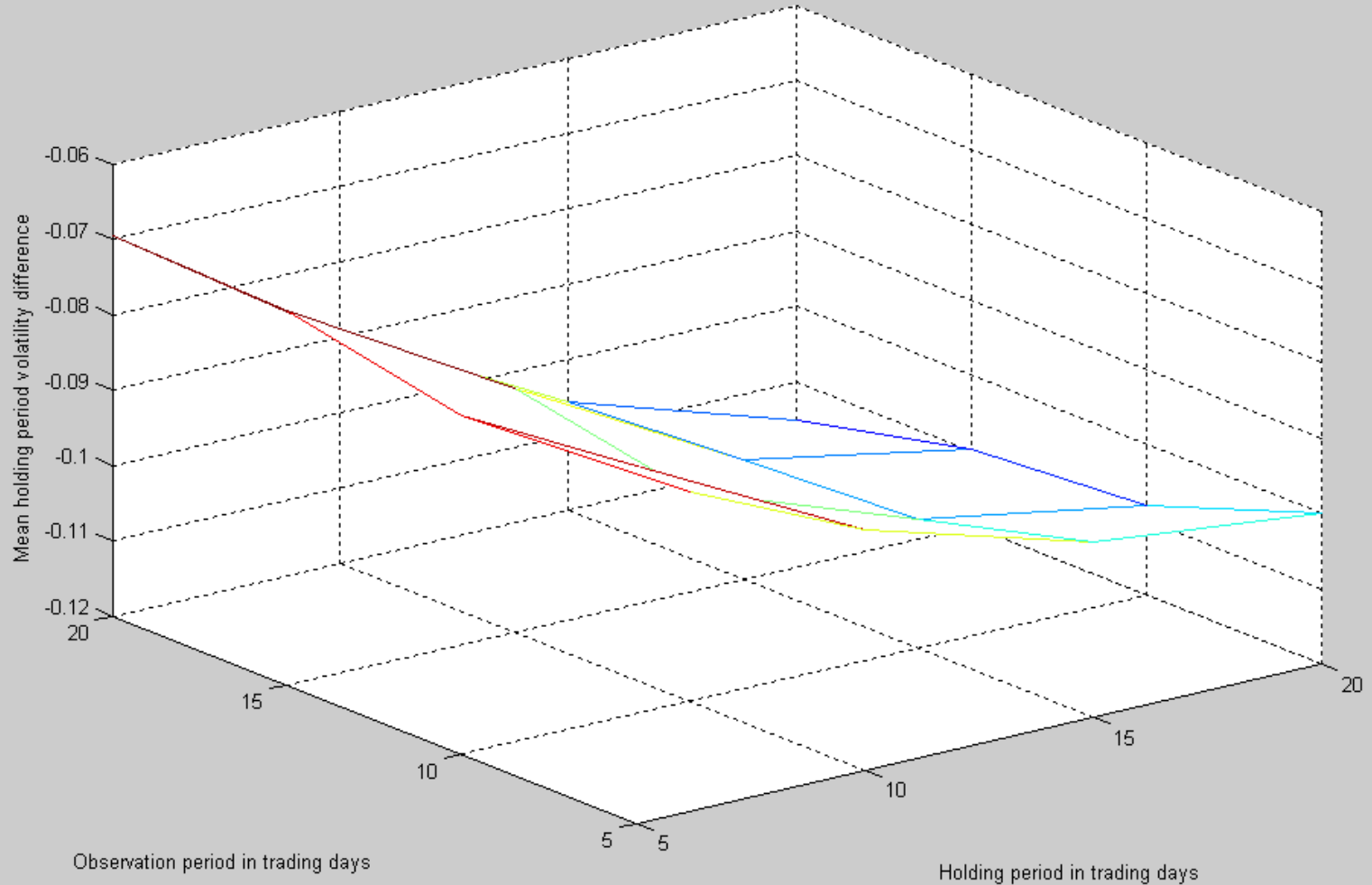
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# Back to Overreaction

- For a given year, each firm now has 4 volatility observations for each populated trading day
- Will form observation-holding period pairs of 1-4 weeks each
- Each trading day, accumulate all firms with enough data to meet a given observation-holding period pair
- Rank by CHANGE in observation period measure implied/garch/excess volatility
- Highest 25 and lowest 25 observation period measures determine accompanying holding period CHANGE measure implied/garch/excess volatility
- Repeat for all possible trading days given length of observation/holding period pair
- Average holding period results over all trading days

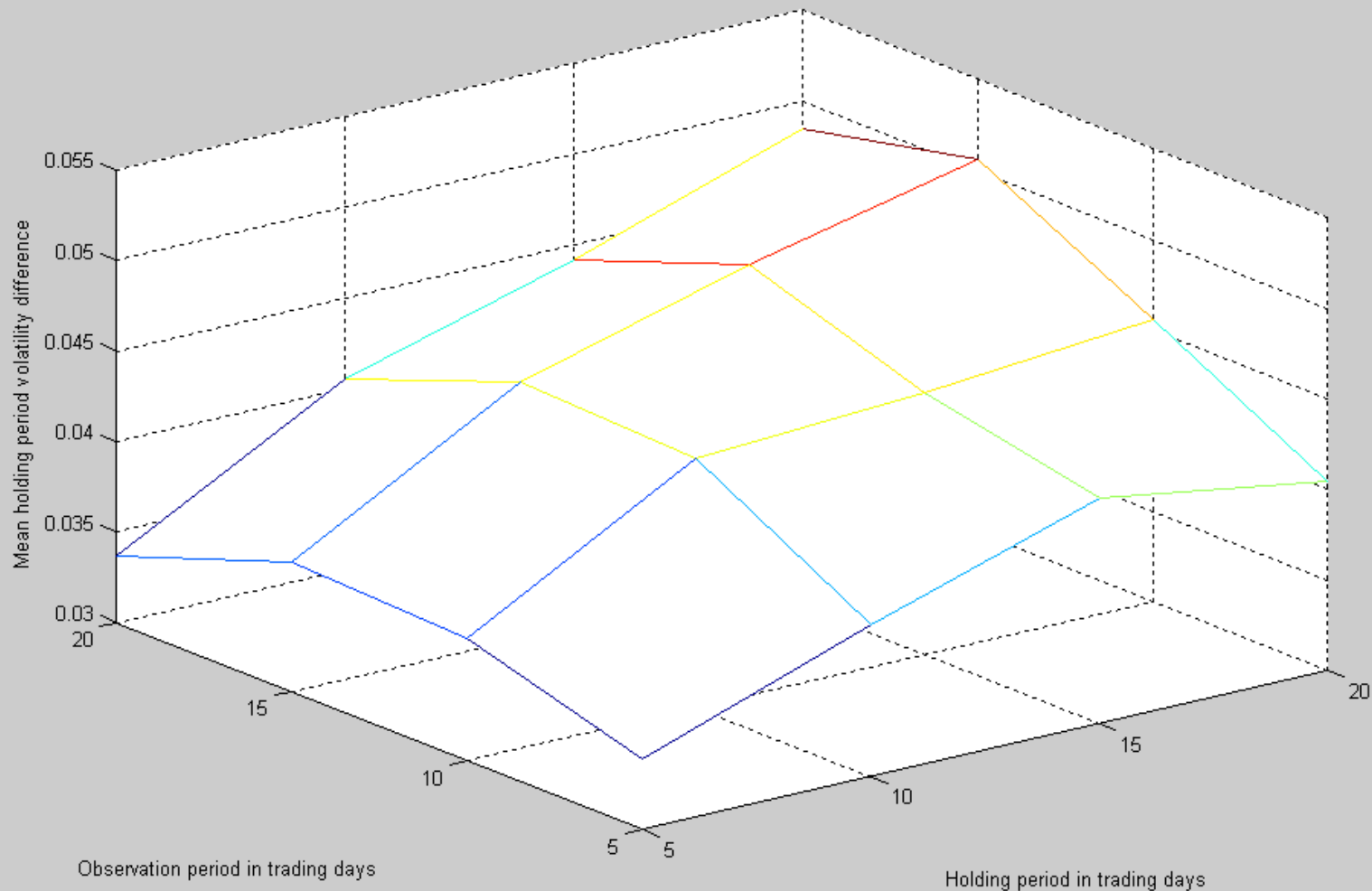
# Overreaction on upside: 30DTM Imp Vol

High Only 30 DTM Implied Obs vs. 30 DTM Implied Hold for 1997 Equity Calls 50 cont. min. Screen 25 firms min/mean/max over OBS-HLD pairs 281/437/670

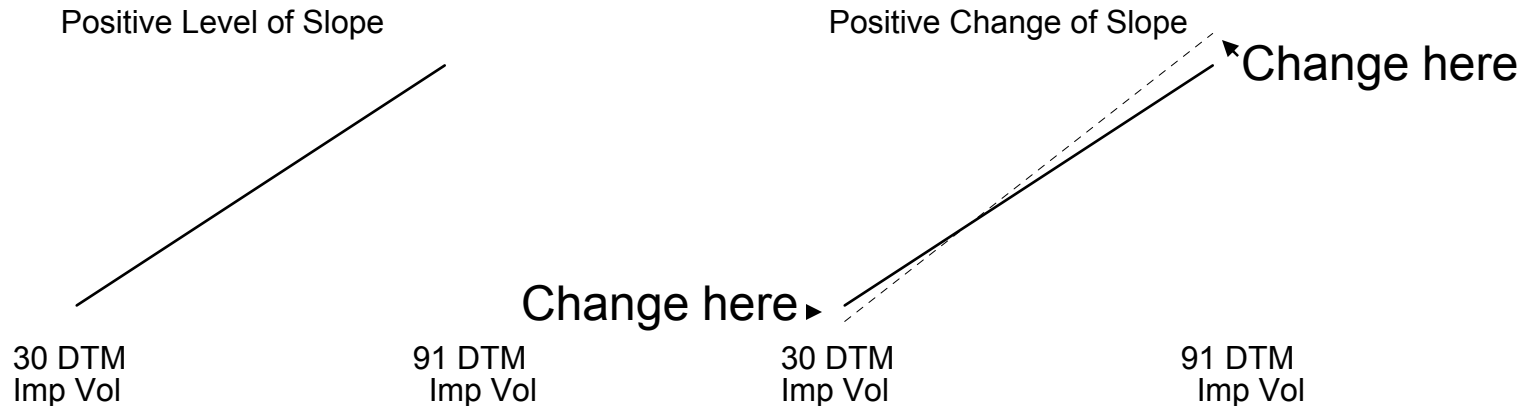


# Overreaction on downside: 30DTM Imp Vol

Low Only 30 DTM Implied Obs vs. 30 DTM Implied Hold for 1997 Equity Calls 50 cont. min. Screen 25 firms min/mean/max over OBS-HLD pairs 281/437/670



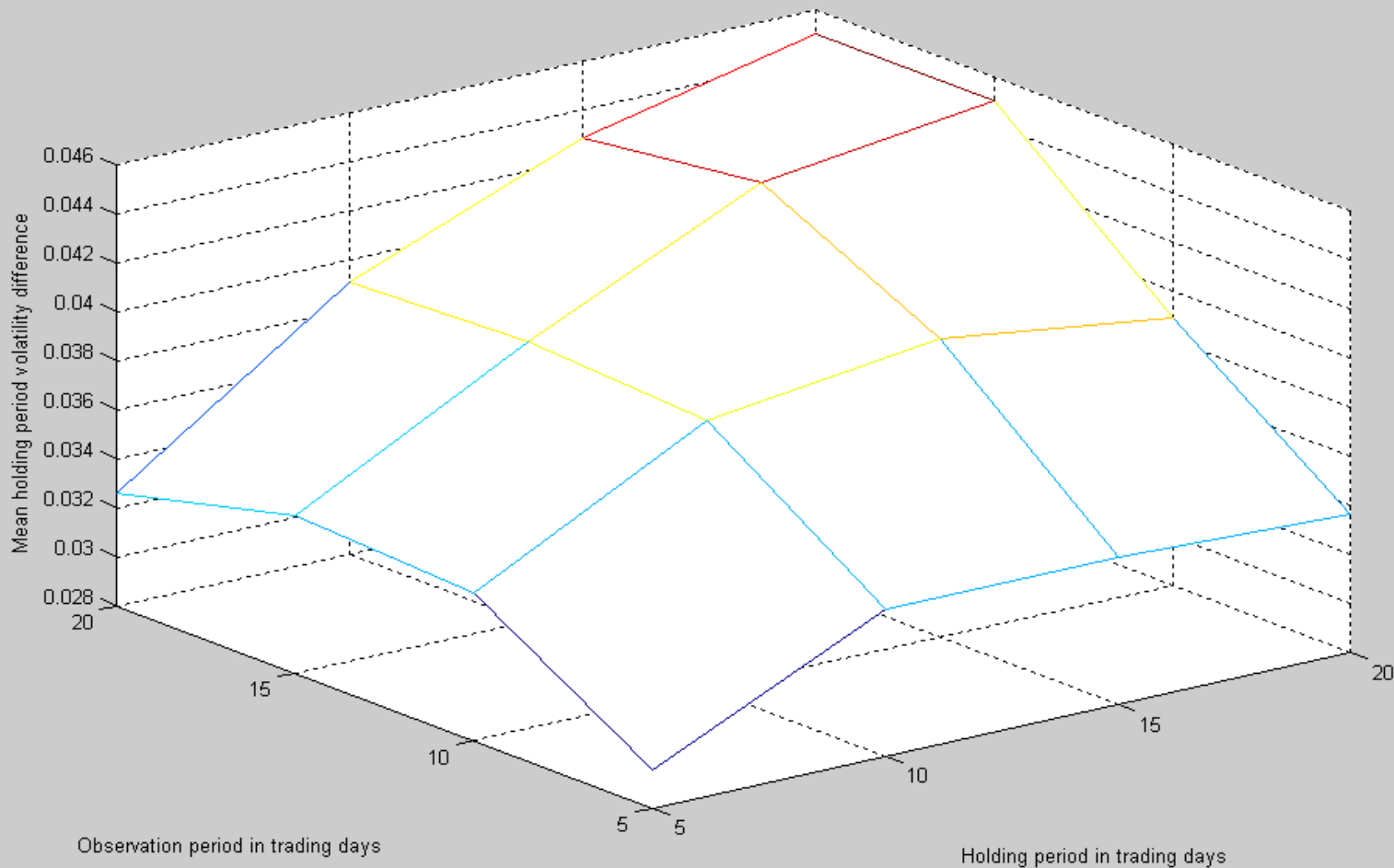
# Envisioning the slope change



- Note that it is the change in the 91DTM minus the change in the 30 DTM not the 91DTM minus the 30 DTM

# Overreaction on downside 2: High implied vol slope change (91DTM change-30DTM change Obs) and 30DTM Hold

High Only 91-30 DTM Implied Obs vs. 30 DTM Implied Hold for 1997 Equity Calls 50 cont. min. Screen 25 firms min/mean/max over OBS-HLD pairs 281/437/670



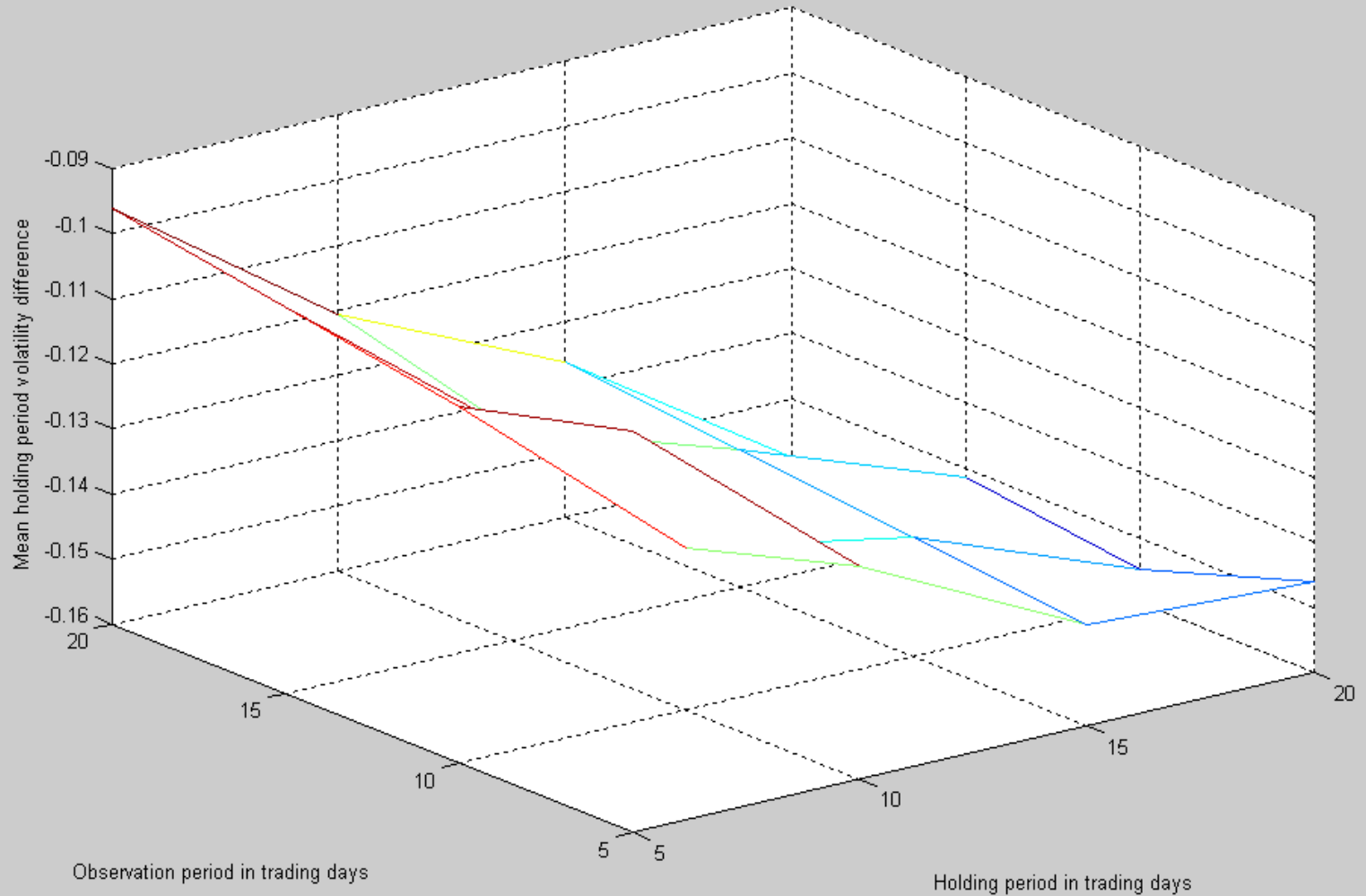
# Overreaction in Implied Vol at a Specific Point

- For an observation period of 20 days and a holding period of 20 trading days we have on average over 400 firms with data each trading day:
  - At the 30DTM point screening the largest 25 implied vol increases (average absolute increase in implied vol of 24%) leads to an average holding period decline of 12% while screening the largest 25 implied vol decreases (23%) leads to an average holding period increase of 5%
  - Looking through the slope measure the largest 25 slope declines (average 16% - from a 91DTM increase of 1% and a 30DTM increase of 17%) leads to a 30DTM average holding period decline of 9% while the largest 25 slope increases (again average 16% - from a 91DTM decrease of 1% and a 30DTM decrease of 17%) leads to a 30DTM average holding period increase of 4%



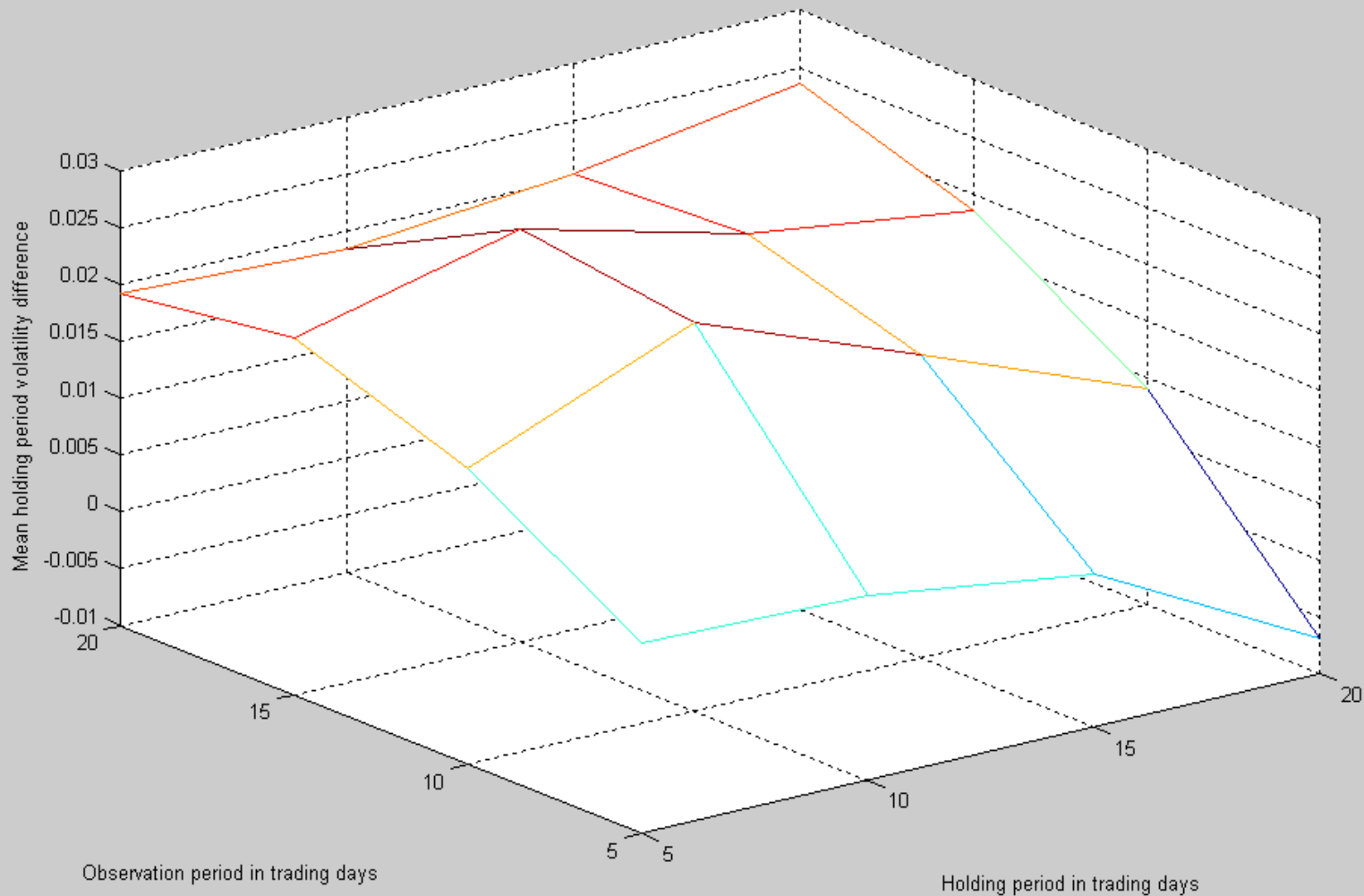
# The Garch Values: Overreaction on upside at 30DTM

High Only 30 DTM Garch Obs vs. 30 DTM Garch Hold for 1997 Equity Calls 50 cont. min. Screen 25 firms min/mean/max over OBS-HLD pairs 281/437/670



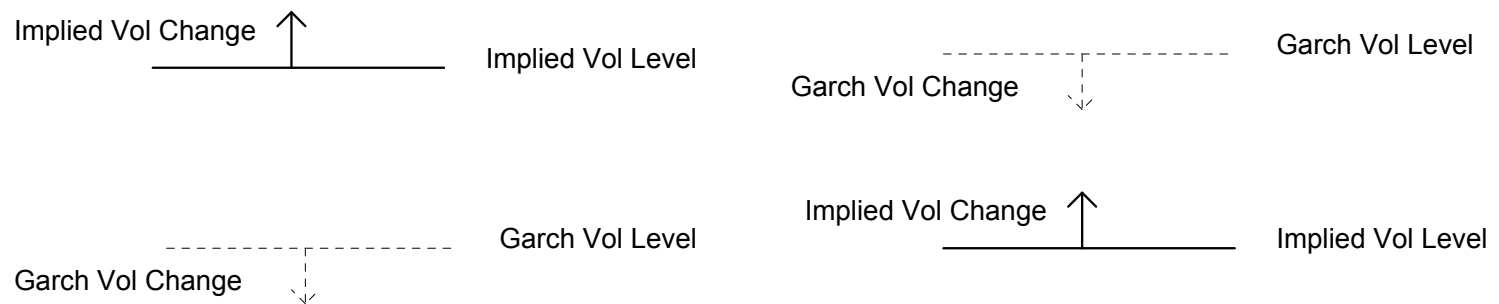
# Overreaction on downside: 30DTM Garch Vol

Low Only 30 DTM Garch Obs vs. 30 DTM Garch Hold for 1997 Equity Calls 50 cont. min. Screen 25 firms min/mean/max over OBS-HLD pairs 281/437/670



# Envisioning Excess Implied Volatility Changes

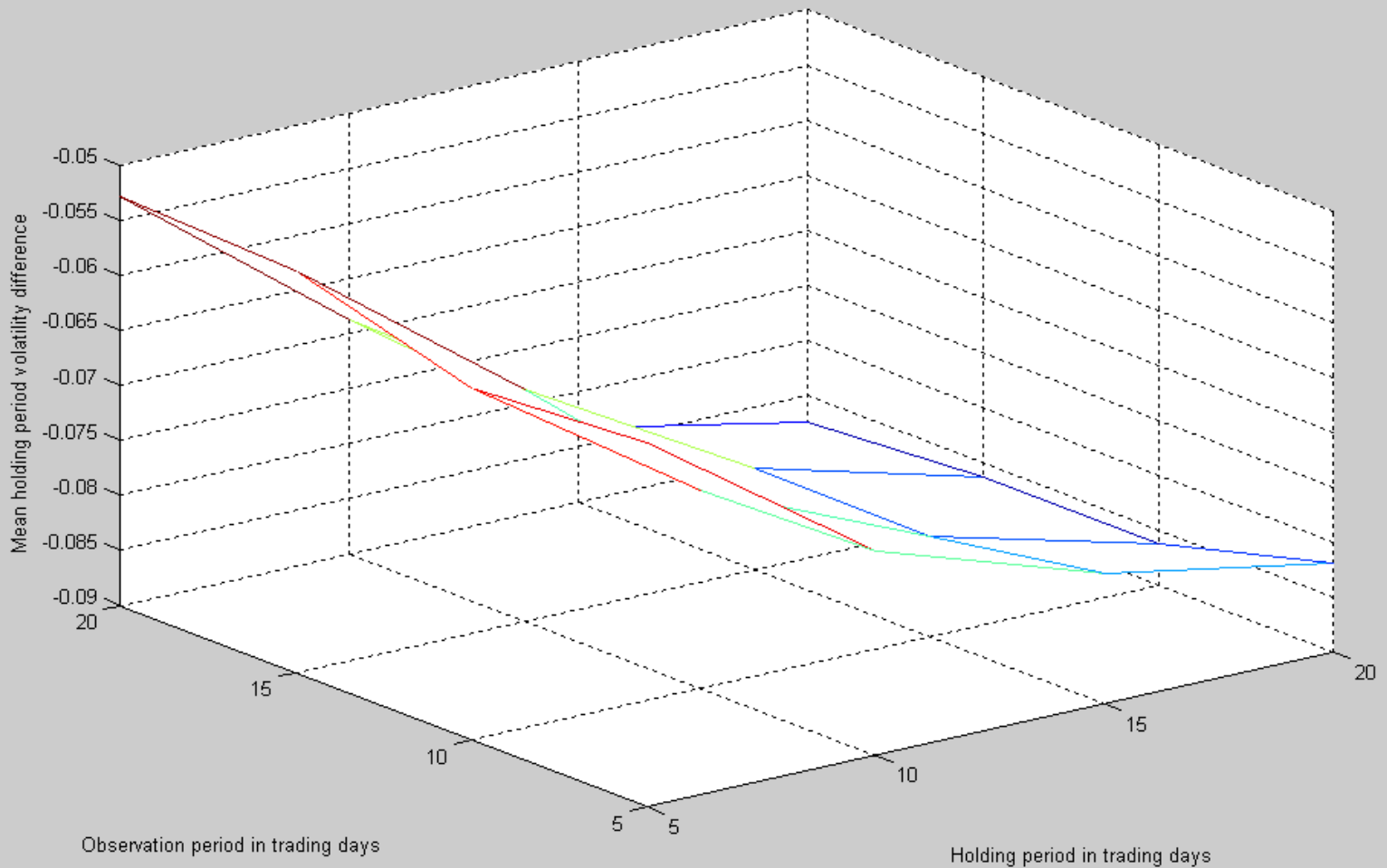
- Note that excess vol level is implied level minus Garch vol level.



- However, we are considering excess vol changes which would be implied change minus Garch vol change (here we see an equivalent increasing change in excess vol)
- We can rank changes in this measure and see what they imply about holding period implied volatility

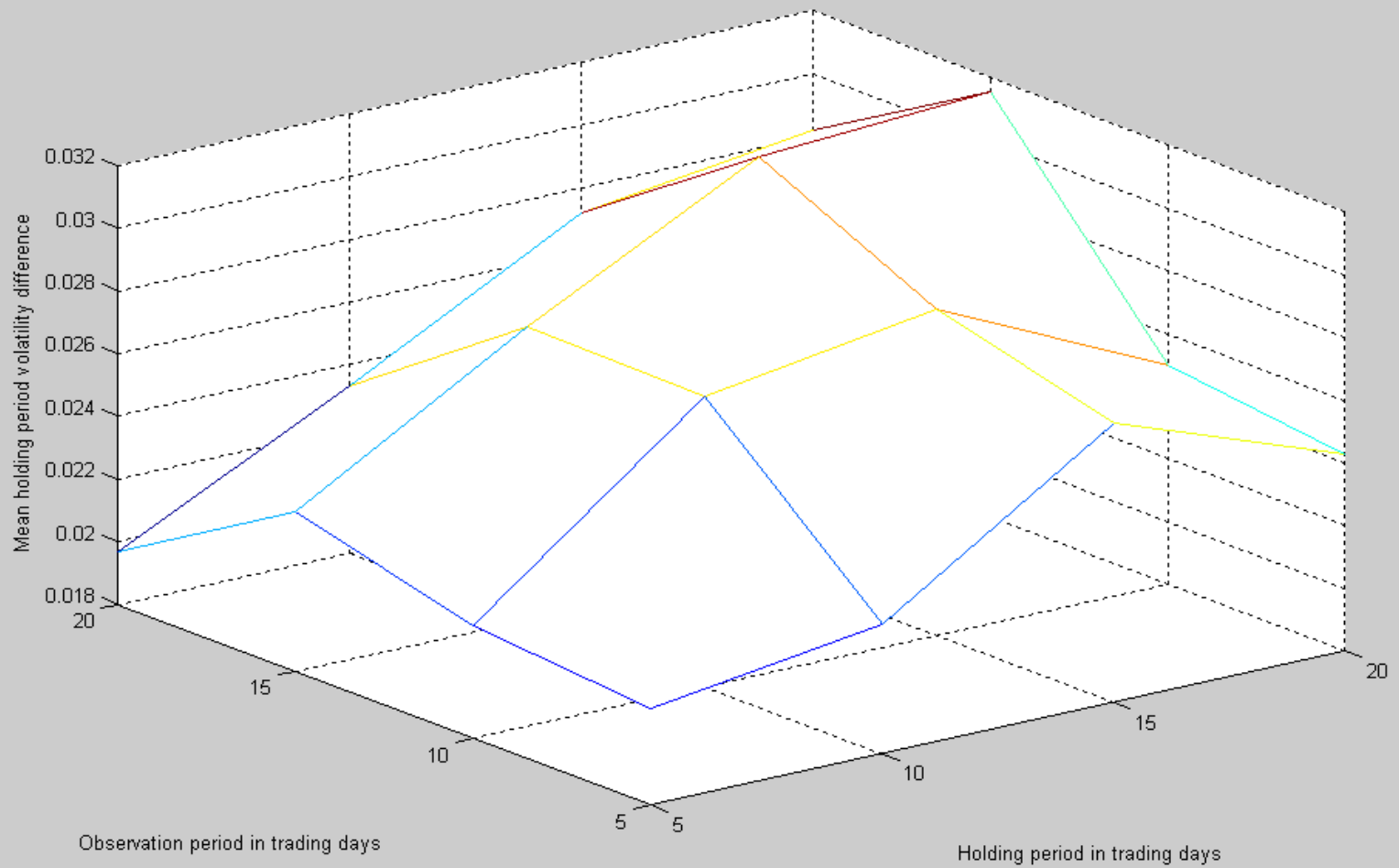
# Excess Implied Volatility: Widening Excess Vol at 30 DTM Produces a decline in Implied Vol at 30 DTM

High Only 30 DTM Excess Obs vs. 30 DTM Implied Hold for 1997 Equity Calls 50 cont. min. Screen 25 firms min/mean/max over OBS-HLD pairs 281/437/670



# Shrinking Excess Vol at 30 DTM Produces increase in Implied Vol at 30 DTM

Low Only 30 DTM Excess Obs vs. 30 DTM Implied Hold for 1997 Equity Calls 50 cont. min. Screen 25 firms min/mean/max over OBS-HLD pairs 281/437/670

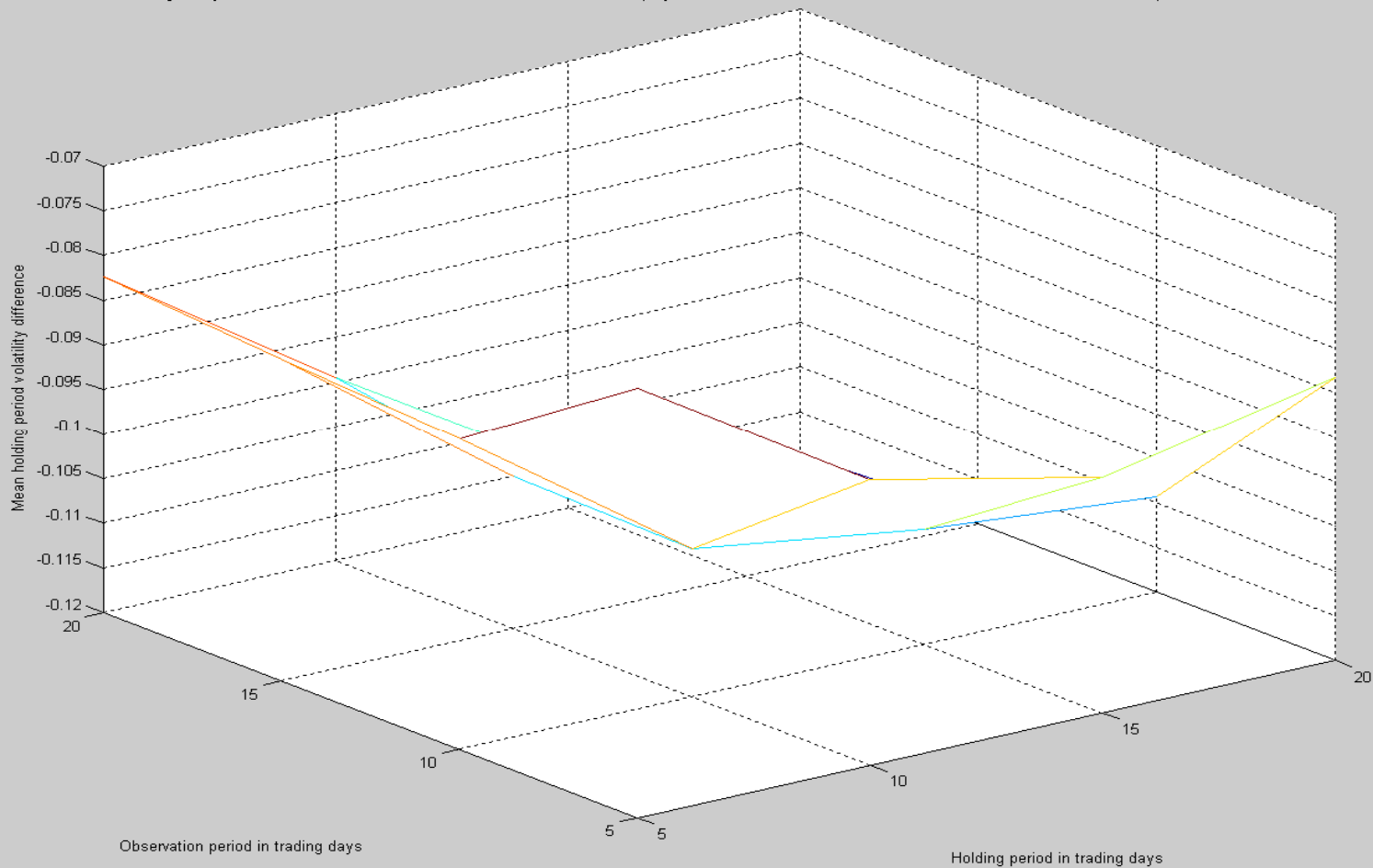


# Excess Vol Change in Observation and Holding Period

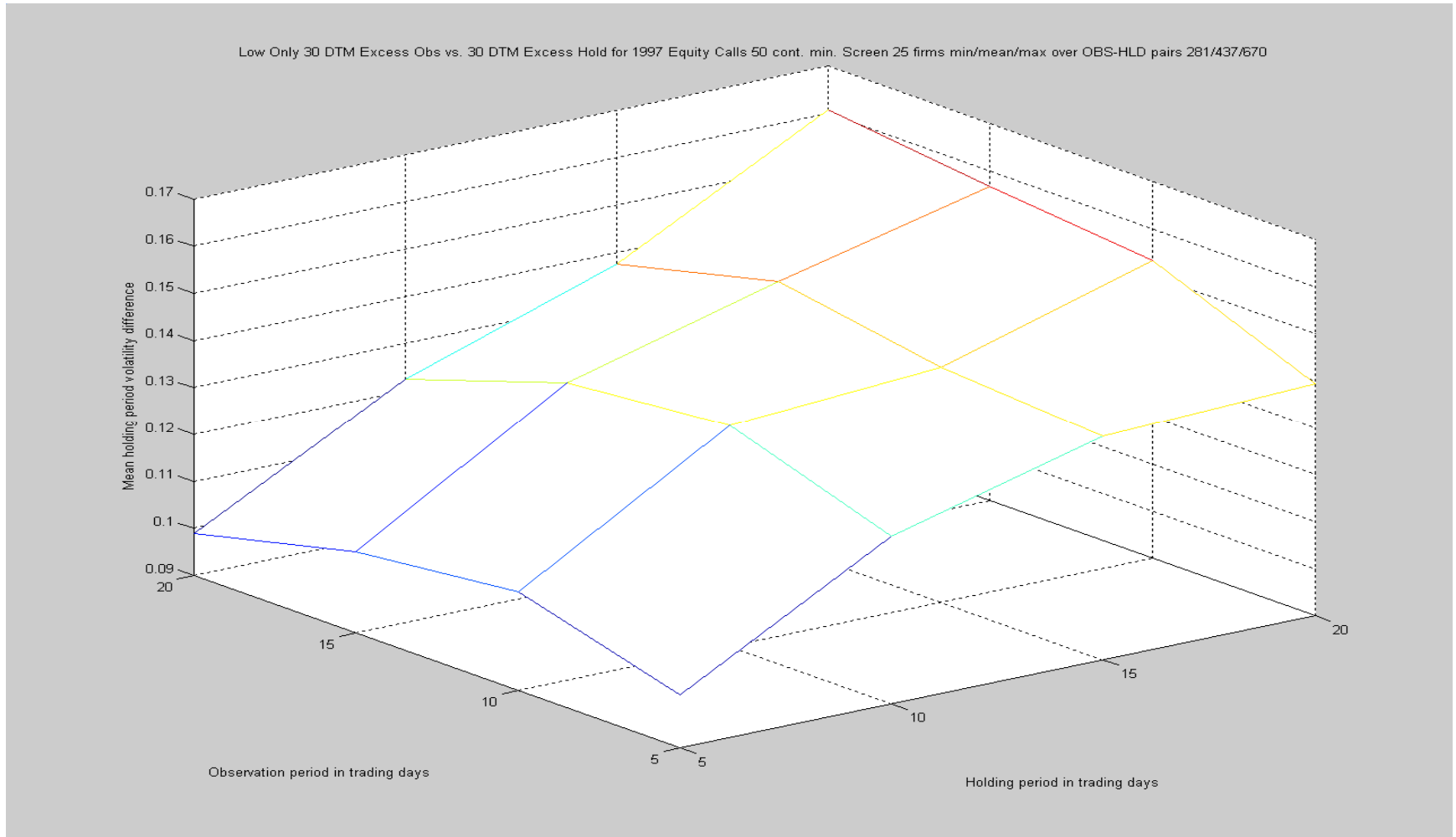
- Excess Vol level could be viewed as a measure of expensiveness
- How do changes in this measure behave, do they overreact?
- We can examine ranked changes in excess volatility and see how large magnitude changes behave during the holding period

# High Excess vol in observation (36% increase over 20 trade days) reverts to lower excess vol in holding period

High Only 30 DTM Excess Obs vs. 30 DTM Excess Hold for 1997 Equity Calls 50 cont. min. Screen 25 firms min/mean/max over OBS-HLD pairs 281/437/670



# Negative change in observation (37% decrease for 20 trade days) leads to increase in holding period excess vol at 30DTM





# 1997 vs. 1998 Single Year

- Implied volatility reverts downward by a similar magnitude but upward reversion at both 30DTM and 91DTM are 1-2 vol points greater in 1998
- Garch volatility reverts downward by 6-10 vol points more and reverts upward by 2-3 vol points more in 1998 than 1997
- Excess volatility reverts downward by 1-2 vol points more and reverts upward by 8-10 vol points more in 1998 than 1997

# 1998 One year fit vs. Two year fit

1998 with trailing returns	2 Year	1 Year
Total Firms	2765	2765
Did not converge	49	18
Data missing or bad fit	262	98
Fitted Firms	2454	2649

- Results of Garch overreaction comparison find that 2 year fit produces 3-8% less overreaction on upside and approximately 1% less overreaction on the downside.

# Garch overreaction comparison for two fits to 1998 data at 20 trade day observation and holding periods

	30 DTM		91 DTM	
	Obs Change	Hold Change	Obs Change	Hold Change
High Results				
1 Year	50.57%	-24.82%	51.38%	-21.16%
2 Years	39.39%	-17.70%	35.95%	-13.01%
Low Results				
1 Year	-39.97%	5.71%	-38.07%	4.69%
2 Years	-30.37%	5.43%	-25.63%	3.81%

# Introduction of Trailing Cumulative Returns

- Working with 1998 data and segregating each set of volatility changes each trading day by cumulative three or twelve month returns (High, Med, and Low) produces the following results
  - High returns have less overreaction in implied volatility on both upside and downside than low returns (losses) at both 30DTM and 91DTM
  - Consistent with higher cumulative returns displaying smaller magnitude changes in implied vol when ranked during observation period (leverage effect)

# Potential Next Steps

- Move to specific options?
  - Examine behavior of specific strategies such as spreads (buy 91DTM, sell 30DTM, etc.)
- More focus upon Garch fitting?
  - Asymmetric models
  - Statistical testing of fitted models
  - Best mix of conditional mean and Garch model to fit a population